

EECS 2030 A

Fall 2017

Advanced Object Oriented Programming

Tuesday September 12

Lecture 2



```
1 public class CircleUtilities {
2     private static final int RADIUS_TO_DIAMETER = 2;
3     static int radius = 10;
4     public static final int PI = 3;
5
6     static int getDiameter() {
7         int diameter = radius * RADIUS_TO_DIAMETER;
8         return diameter;
9     }
10    static int getDiameter(int radius) {
11        return radius * RADIUS_TO_DIAMETER;
12    }
13    static void setRadius(int newRadius) {
14        radius = newRadius;
15    }
16    public static int getCircumference(int radius) {
17        return getDiameter(radius) * PI;
18    }
19    public static int getCircumference1() {
20        return getDiameter() * PI;
21    }
22    private static int getCircumference2() {
23        return getCircumference(radius);
24    }
25 }
```

attributes

methods

variable

no inputs

one input

methods :

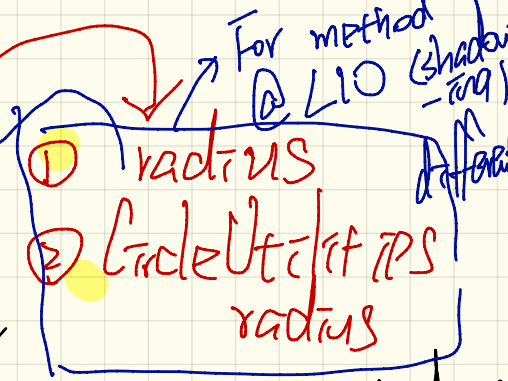
ACCESSOR - return non-void

MUTATOR - return void

We overload the method getDiam. with different parameter lists

```
1 public class CircleUtilities {
2     private static final int RADIUS_TO_DIAMETER = 2;
3     static int radius = 10;
4     public static final int PI = 3;
5
6     static int getDiameter() {
7         int diameter = radius * RADIUS_TO_DIAMETER;
8         return diameter;
9     }
10    static int getDiameter(int radius) {
11        return radius * RADIUS_TO_DIAMETER;
12    }
13    static void setRadius(int newRadius) {
14        radius = newRadius;
15    }
16    public static int getCircumference(int radius) {
17        return getDiameter(radius) * PI;
18    }
19    public static int getCircumference1() {
20        return getDiameter() * PI;
21    }
22    private static int getCircumference2() {
23        return getCircumference(radius);
24    }
25 }
```

For method @ L10 same shadowing



due to shadowing, if you want to refer to radius @ L3 => CircleUtilities.radius

```
1 public class CircleUtilities {
2     private static final int RADIUS_TO_DIAMETER = 2;
3     static int radius = 10;
4     public static final int PI = 3;
5
6     static int getDiameter() {
7         int diameter = radius * RADIUS_TO_DIAMETER;
8         return diameter;
9     }
10    static int getDiameter(int radius) {
11        return radius * RADIUS_TO_DIAMETER;
12    }
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14        radius = newRadius;
15    }
16    public static int getCircumference(int radius) {
17        return getDiameter(radius) * PI;
18    }
19    public static int getCircumference1() {
20        return getDiameter() * PI;
21    }
22    private static int getCircumference2() {
23        return getCircumference(radius);
24    }
25 }
```

CU. getCircumference()

helper methods

A method is a block of code which can be reused by referring to its name.

shadowing

```

1 public class CircleUtilities {
2     private static final int RADIUS_TO_DIAMETER = 2;
3     static int radius = 10;
4     public static final int PI = 3;
5
6     static int getDiameter() {
7         int diameter = radius * RADIUS_TO_DIAMETER;
8         return diameter;
9     }
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11        return radius * RADIUS_TO_DIAMETER;
12    }
13    static void setRadius(int newRadius) {
14        radius = newRadius;
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16    public static int getCircumference(int radius) {
17        return getDiameter(radius) * PI;
18    }
19    public static int getCircumference1() {
20        return getDiameter() * PI;
21    }
22    private static int getCircumference2() {
23        return getCircumference(radius);
24    }
25 }

```

CU. getCircumference(5)

= CU. getParameter(5) \* 3

= 5 \* 2 \* 3

Argument

value

for replacing

parameter

radius

parameter

parameter

fac(x) = x \* (x-1) \* x

fac(5) fac(4) arguments :- 1

# modifiers

1. Visibility

private  
package  
class

public

you must access this class name.

variable

private

2. Constant/Variable

double

myPI = 3.14 ; ✓  
myPI = 6.28 ;

final static int Foo = 2

final double PI = 3.14 ;

~~Foo = 4 ;~~

~~PI = 6.28 ;~~  
... it's constant

```
class MyMath {
    public static int Foo = 3;
}
```

↓  
 There's guaranteed only one copy of Foo will exist at runtime

We can only use the name of class to access this attribute 'cause it's static

MyMath.Foo

MyMath	
Foo	3

```
class MyMathUser {
    ...
    main( ) {
        MyMath.Foo = 4;
    }
}
```

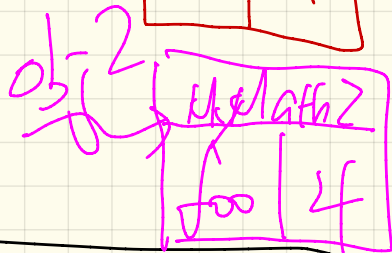
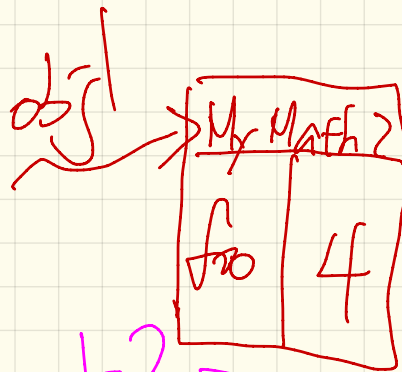
```
class MyMath2 {
```

```
public int foo = 4
```

4

non-static  
attribute

```
}
```



obj1.foo  
obj2.foo

```
class MyMath2App {
```

```
main() {
```

```
MyMath2
```

obj1 =

new

```
MyMath2();
```

```
MyMath2
```

obj2 =

new

```
MyMath2();
```

```
}
```

```
}
```

~~MyMath2.foo~~

ambiguous

∴ two copies  
of MyMath2

Thursday Sept. 14

Lecture 3



# office hours

13:30 - 15:30

M, Tu, Th.

# Number division in Java

$$\boxed{\phantom{00}}_1 / \boxed{\phantom{00}}_2$$

If both  $\boxed{\phantom{00}}_1$  and  $\boxed{\phantom{00}}_2$   
are integers, the result

is quotient.

Otherwise, result with precision.

print(8 / 2)     4     (2 + 3) \* 4  
print(8.0 / 2)     4.0

print(9 / 2)     4  
print(9 / 2.0)     4.5

step 2:  
9.0 / 2  
= 4.5

int i = 9;  
int j = 2;

→ print(i / j) 4

① ~~4.5~~ print((double) i / j)     <sup>step 1 9.0</sup>

② ~~4.5~~ print(((double) 9.0) / j)

2 / 3 0

2.0 / 3

---

$\overline{m}t$  /  $\overline{m}t$   
 $\overline{c}$  /  $\overline{j}$

großartig

(double)  $\overline{c}/\overline{j}$

X

~~$\overline{c}.0$  /  $\overline{j}$~~

(double)  $\overline{c}/\overline{j}$  ✓

((double)  $\overline{c})/\overline{j}$  ✓

```
1 public class CircileUtilitesApplication {
2     public static void main(String[] args) {
3         System.out.println("Initial radius of CU: " + CircleUtilities.radius);
4         int d1 = CircleUtilities.getDiameter();
5         System.out.println("d1 is: " + d1);
6         System.out.println("c1 is: " + CircleUtilities.getCircumference1());
7         System.out.println("=====");
8         System.out.println("d2 is: " + CircleUtilities.getDiameter(20));
9         System.out.println("c2 is: " + CircleUtilities.getCircumference(20));
10        System.out.println("=====");
11        System.out.println("Change the radius of CU to 30...");
12        CircleUtilities.setRadius(30);
13        System.out.println("=====");
14        d1 = CircleUtilities.getDiameter();
15        System.out.println("d1 is: " + d1);
16        System.out.println("c1 is: " + CircleUtilities.getCircumference1());
17        System.out.println("=====");
18        System.out.println("d2 is: " + CircleUtilities.getDiameter(20));
19        System.out.println("c2 is: " + CircleUtilities.getCircumference(20));
20    }
21 }
22
```

```

1 public class CircleUtilities {
2     private static final int RADIUS_TO_DIAMETER = 2;
3     static int radius = 10;
4     public static final int PI = 3;
5
6     static int getDiameter() {
7         int diameter = radius * RADIUS_TO_DIAMETER;
8         return diameter;
9     }
10    static int getDiameter(int radius) {
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19    public static int getCircumference1() {
20        return getDiameter() * PI;
21    }
22    private static int getCircumference2() {
23        return getCircumference(radius);
24    }
25 }

```

print(CU.radius) 10  
 int d1 = CU.getDiameter();  
 println(d1); 20  
 println(CU.getDiameter(20)) 60  
 println(CU.getDiameter(20)) 40  
 println(CU.getCirc(20)) 120

CU	
R-T-D	2
PI	3
radius	10
:	

$20 \times 2 \times 3$   
 $20$   
 $d1 = \text{int}$

~~CUt:App~~ }  
client

~~CUt:1~~  
↓  
supplier

get Area (~~radius~~<sup>-10</sup>)  
↓  
service  
name

supplier will  
just calculate  
 $(-10)^2 \times 3 = 300$   
not good  
should be  
an error

}

```
class MyUtil {
```

```
    static void m(int x) {
```

```
        . . .
```

```
    }
```

```
}
```

```
class MyUtilApp {
```

```
    MyUtil.m(23.4);
```

(int) 23.4

sample

.7



Counter :

$$0 \leq \text{value} \leq 3 \quad \text{Req.}$$

void increment ( ) {

~~Req.~~ : When should we throw a

IllegalArgument Exception ?

value ++ ;

① value >  $\frac{\text{MAX}}{3}$  ]

inappropriate  
if value = MAX

Section B

Guest Lecture I

MAX = 3      MID = 0

word

increment ( )  
if ( ~~counter~~ > MAX ) {

assume counter is currently MAX

throw \_\_\_\_\_

}

else {

← reaching this point means:

counter ++ ;      !(counter > MAX)

}

≡ counter ≤ MAX

# Lifetime of a counter

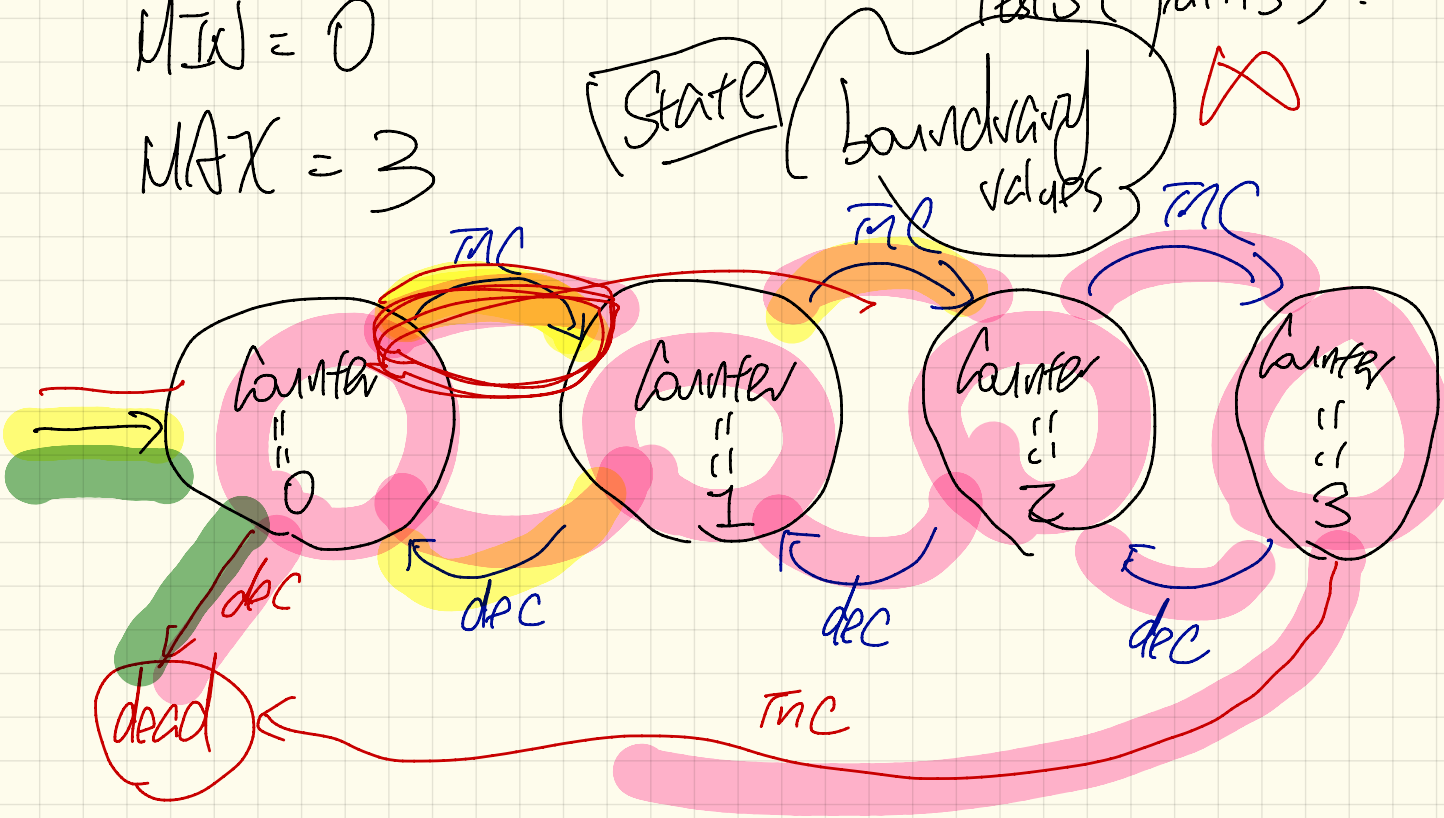
Q. How many tests (paths)?

MIN = 0

MAX = 3

State

boundary values



Correctness

efficiency

of your code

my code is "correct"  
if all scenarios have been tested and passed, to my best knowledge

```
class Counter {
```

```
    MIN
```

```
    MAX
```

```
    static Counter = 3; }
```

```
    increment() { -- }
```

```
    decrement() { -- }
```

```
}
```

Supplier

Unit tests are  
use cases of  
Counter.

```
class TestCounter {
```

```
    @Test
```

```
    void testI() {
```

```
        int v = Counter.counter;
```

```
        Assert Assert.assertTrue(  
            3 == 0);  
    }
```

```
}
```

Client

September 19

Tuesday

Lecture 4

② X e.g. counter == 1 | 1 <= 3 → T

↳ exception is thrown.

increment (int value) {

else: ← counter <= MAX

~~① counter > MAX~~  
~~② counter <= MAX~~  
~~③ counter >= MAX~~  
throw

← counter < MAX  
else {

③ ✓

≠ counter + 1 ; ①

}  
}

① X ∴ if counter == MAX  
MAX > MAX → False



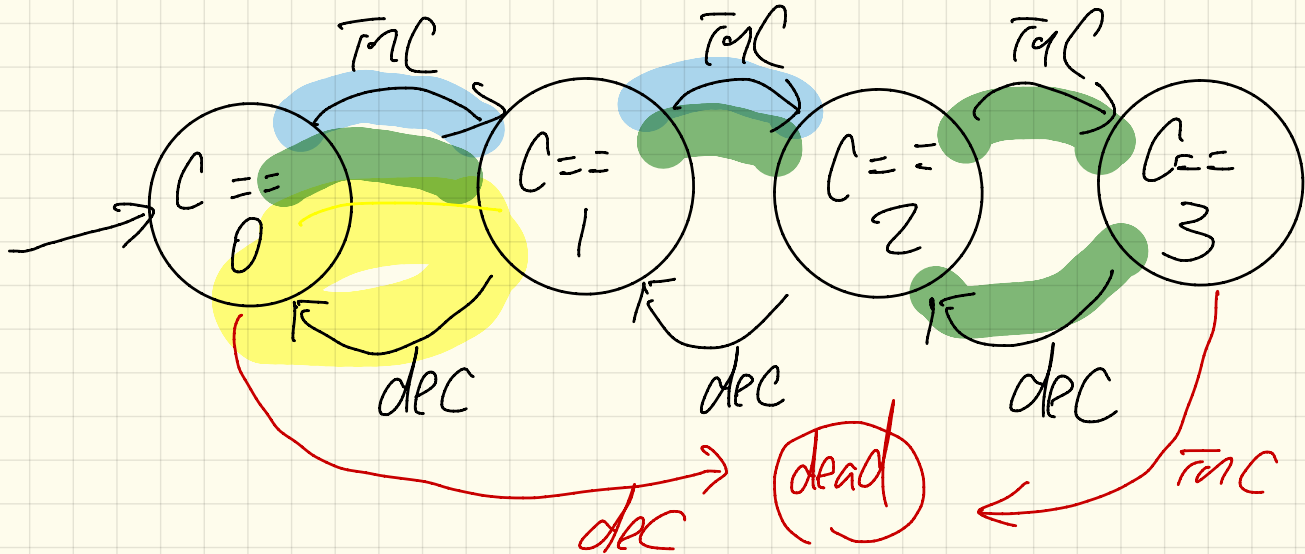
# State Machine

How many test cases?  
How many paths?

TRANSITIONS

(mutator method calls)

[state] values of attributes



# Criteria for Judging your program

① Correctness

② Efficiency.

Opt A. ①  $\wedge$   $\neg$ ②

Opt B.  $\neg$ ①  $\wedge$  ②

→ define a reasonably complete set of TESTS -

```
class Counter {
```

```
    inc(int v) {
```

```
        --
```

```
    }
```

```
}
```

supplier

```
class TestCounter {
```

```
    @Test
```

```
    void testInc() {
```

```
        counter.inc();
```

```
        assertTrue()
```

```
            counter.value == 1
```

Client: JUnit class

Now we want to test an abnormal use of the class, for which we expect a precondition violation to occur.

@Test

```
void testDecFromZero() {
```

try {

counter.value = MIN;  
// mit val ist 0\*

counter.dec(); // expect exception/

→ // We do not expect to reach this line //  
// fail C "no precondition violation occurred"

```
catch (IAE e) {
```

```
} } // precondition violation occurred as expected //
```

breakpoints and

debugger

↳ slow down your  
program and  
execute line by line.

Counter	
Counter	<del>0</del> 1
MIN	0
MAX	3

① testOneIncrement()

② testDecFromZero()

When starting each test method we should reset the attributes to their initial values to make sure it's a fresh start.

Wednesday Sept. 20

Guest Lecture 2

When does a test method succeed?  
@Test

- No exceptions thrown  
- No assertions failures } Success

Otherwise (exception or assertion failure) } failure



For utility classes, where everything is static, the order in which you run the test methods matters.

### Visualizing a UC

test Increment Twice

increment 0 → 1

1 → 2

test Increment Once

↳ starting value of counter is no longer 0 → (2)

Counter	
MIN	0
MAX	3
value	0 1 2

# Test for abnormal scenario

@Test

```
void testDecFromZero() {
```

```
try {
```

```
    counter.value = MIN;
```

expect  
IAE  
thrown

```
    counter.decrement();
```

```
    fail("IAE not thrown") ← IAE not thrown  
                                → fail
```

```
    catch (IAE e) {
```

```
        * IAE thrown as expected, do nothing */
```

```
    }
```

breakpoints

debugger

↳ put program execution  
in slow motion  
so you can examine  
variables in intermediate  
steps.

class vs. object  
template instance

```
class Point {  
    int x;  
    int y;  
}
```

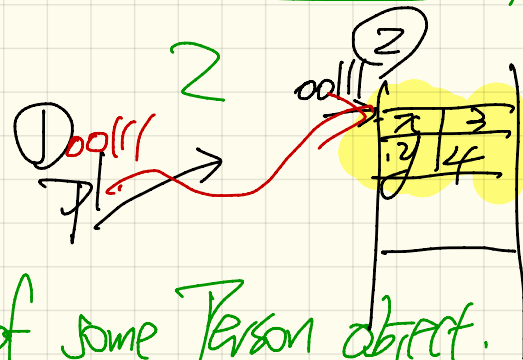
template:  
every Point instance must have x and y.

```
class PointApp {  
    main(...) {  
  
        Point p1 = new Point  
                    (2, 3);  
  
        Point p2 = new Point  
                    (3, 4);  
  
    }  
}
```

$\text{Point } p1 = \text{new Point}(3, 4);$

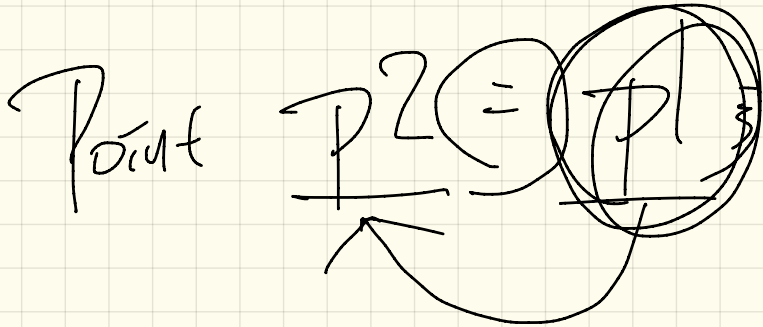
1                      3

- ① Declare a variable  $p1$ .  
 $p1$  can store the address of some Person object.
- ② Allocate space in memory and create a Point with  $x == 3, y == 4$ .
- ③ Store the address of the new Point object in  $p1$ .

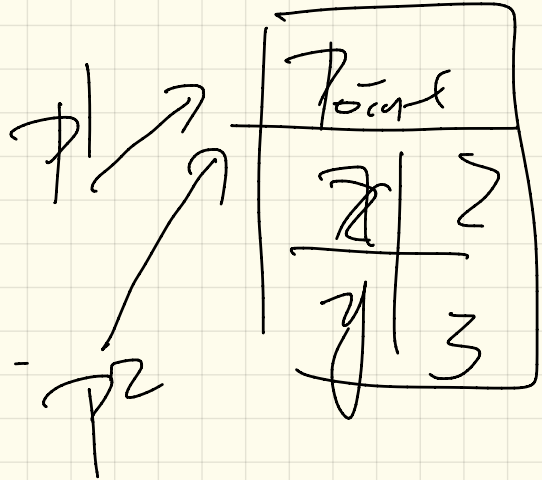


Point p1 = new Point(2, 3);

Point p2 = p1

A diagram illustrating pointer assignment. It shows the text "Point p2 = p1". The "p1" on the right is circled with multiple overlapping lines. An arrow points from this circled "p1" to the "p2" on the left.

aliasing



Thursday Sept. 21

Lecture 5

Why OO

→ template

Observe

P1  
• (x<sub>1</sub>, y<sub>1</sub>)  
2 3

P2  
• (x<sub>2</sub>, y<sub>2</sub>)  
2 -3

Model

```
class {  
    Point  
  
    int x;  
    int y;  
}
```

Execute

P1 → 

Point
x   2
y   3

P2 → 

Point
x   2
y   -3



p1      p2

context  
↑  
object

p1.getY()

method

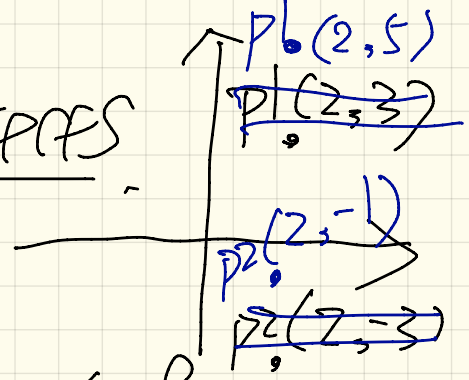
dot notation

p1.moveUp(2)

p2.getY()

p2.moveUp(2)

point objects



→

calling the same method on different contexts may give you different results.

→

Given some natural language

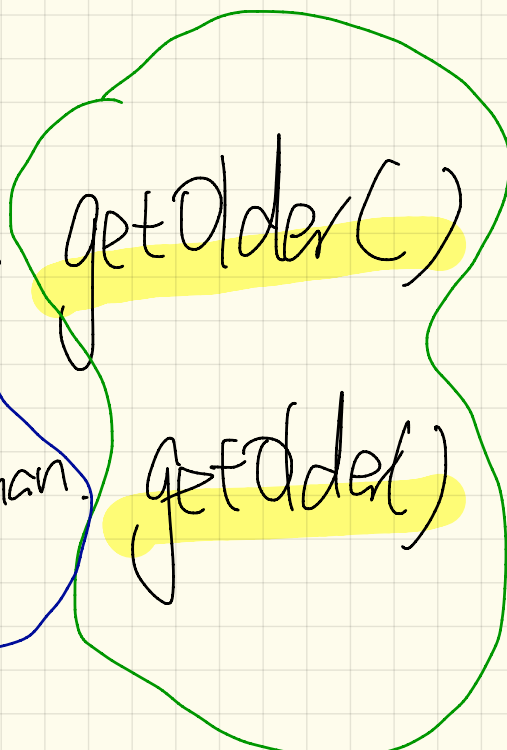
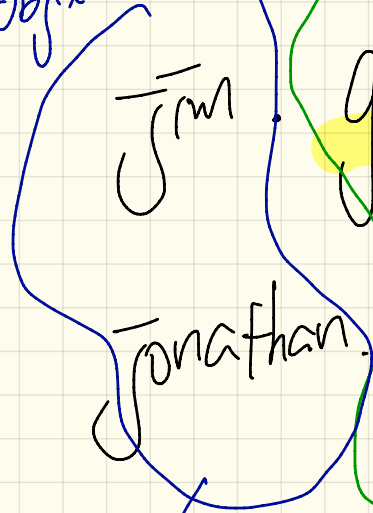
requirements,

we can:

1. Identify nouns either classes or attributes
2. Identify verbs as methods.

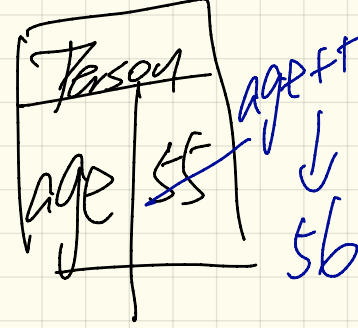
change attributes  
ask something  
about attributes

Context  
objects

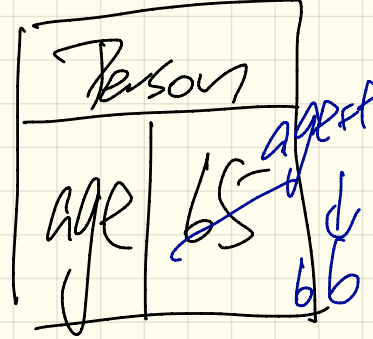


attribute  
values are  
different

Jim →



Jonathan →



These two method calls  
share the same definition:  
age + 1;

# default values

When uninitialized, variables can be assigned their default values

Primitive  
type

int	}	0
long	}	0
float	}	0.0
double	}	0.0
boolean	}	false

Reference  
type

String	}	<u>null</u>
--------	---	-------------

no address  
of string object being  
stored.

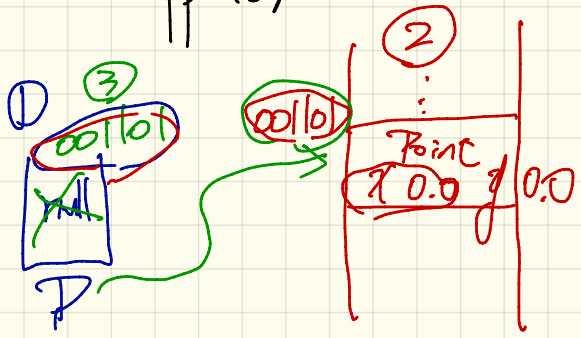
```

class Point {
    double x;
    double y;
    Point () {
        // default values
    }
}

```

③ Store 00101 into p.

Supplier



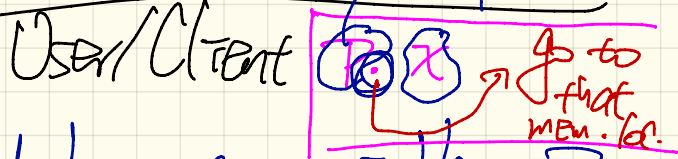
```

class PointApp {
    main(-) {
        Point p = new Point();
    }
}

```

①      ③      ②

→ 00101



- ① We declare a variable P. P can only store addresses of a memory portion that store Point information. Point info.
- ② Allocate a memory portion for storing

```

class Point {
    double x = 6      8
    double y = 3      4 ← define
    Point (double x, double y) {
        pl this. x = x;
        p2 this. y = y;
    }
}

```

Point	
x	3
y	4

p1

p2

Point	
x	6
y	8

```

Point p1 = new Point(3, 4);

```

Context object

```

Point p2 = new Point(6, 8);

```

Context object

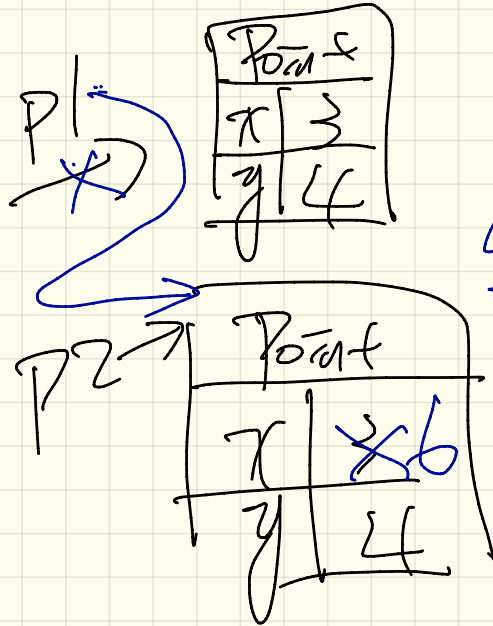
Point p1 = new . . .

Point p2 = new . . .

p1 = p2 ;

p1.x = 6

p2.x ?



int i = 3;    [3]    [4]  
int j = 4;    i    j

i == j    F

Point p1 = new Point(3,4)    ~~[3,4]~~    [3,4]

Point p2 = new Point(3,4)    p1    p2

p1 = p2,    p1 == p2



Lecture 6

Tuesday Sept. 26

```

class Point {
    double x;
    double y;

```

```

    void moveUp(int u) {

```

```

        p1 this.y += u;
        p2 this.y += u;
    }

```

```

    Point(double x, double y) {

```

```

        this.x = x;
        this.y = y;
    }

```

```

class PointTester {

```

```

    main() {
        Point p1 = new Point(3, 4);

```

obj p1

```

        Point p2 = new Point(6, 8);

```

```

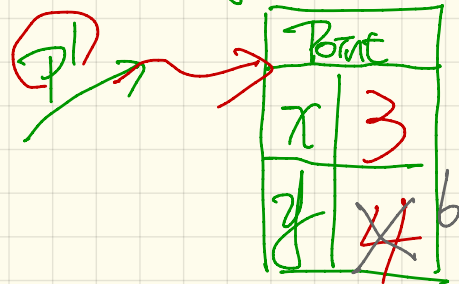
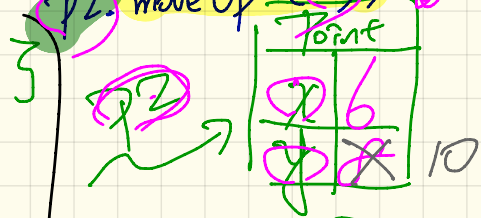
        p1.moveUp(2);

```

```

        p2.moveUp(2);
    }

```



Print p1 = (new) Point (3, 4);

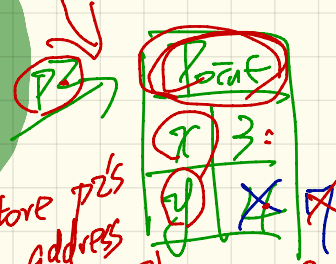
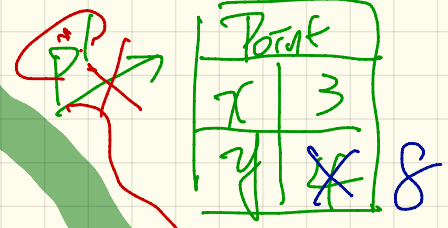
Print p2 = (new) Point (3, 4);

print(p1.x)    print(p1.y)    p2.x    p2.y  
3                    4                    3                    4

p1.moveUp(4);

p2.moveUp(3);

p1.x    p1.y    p2.x    p2.y  
3                    8                    3                    7



Store p2's address into p1  
p1 = p2

p2.moveUp(2) <sup>12</sup>

p1.moveUp(3)

p1.x    p1.y    p2.x    p2.y  
3                    7                    3                    12

Point p1 = new Point(3, 4);

Point p2 = new Point(6, 8);

p1 = p2;

p2 = p1;

p2.moveUp(3);

p1.moveUp(2);

p1.x == p2.x  
p1.y == p2.y  
10

Point	
x	3
y	4

p1

p1.x == p2.x  
p1.y == p2.y  
13

Point	
x	6
y	8

```
class Point {
    double x;
    double y;
}
```

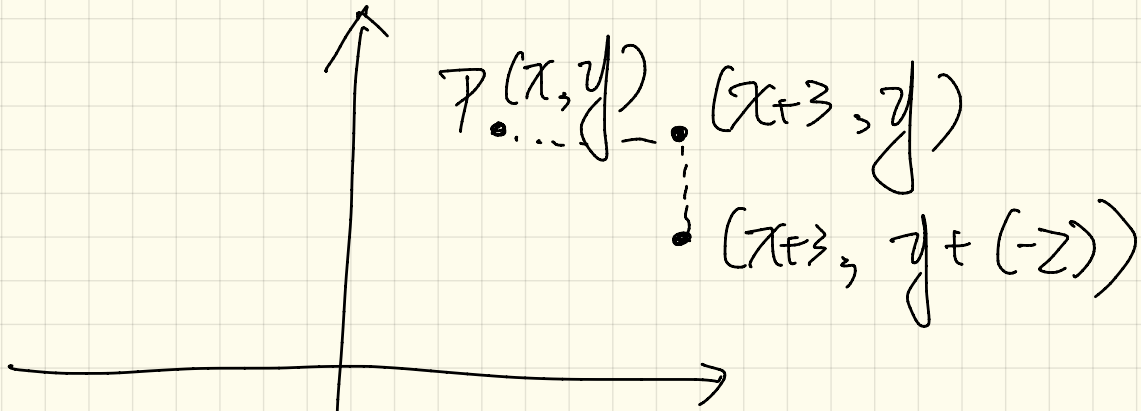
Point p = new Point(5, 3);

wrong  
shadowing

```
Point (double x, double y) {
    x = x;
    y = y;
}
```

right

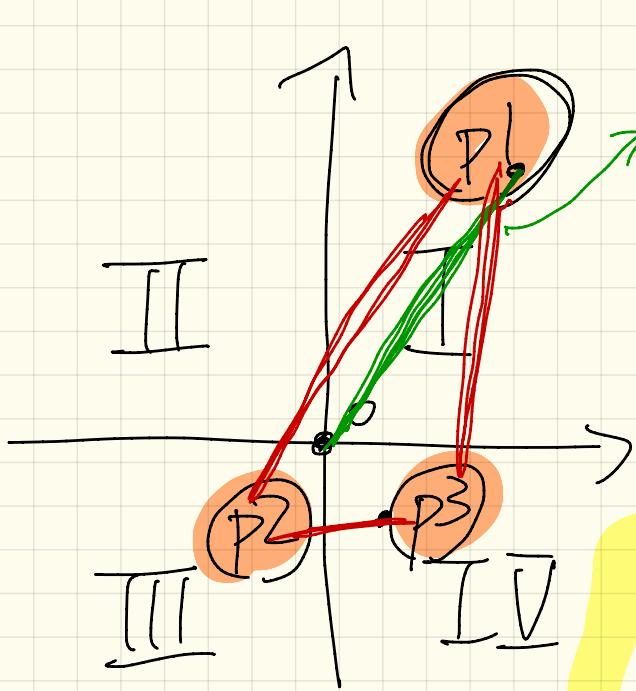
```
Point (double x, double y) {
    this.x = x;
    this.y = y;
}
```



P. move (0, 3, 0)

Ⓟ P. move (  $\frac{3}{\downarrow}$  ,  $\frac{-2}{\downarrow}$  )

horizontal      vertical



$p1.getV(p2)$

$p1.isAtFirstQuadrant()$

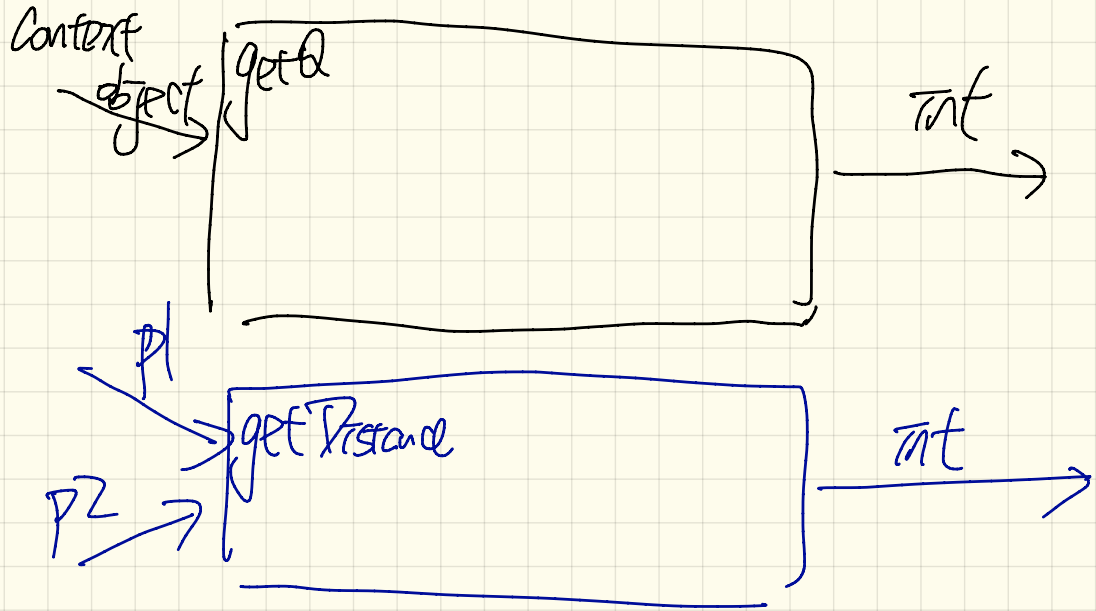
$p2.isAtFQ()$

$p1.getDistance(p2)$

$p1.getDistance(p2, p3)$

$p1.getQ()$	return	1
$0.getQ()$	.	0
$p3.getQ()$		4

Method - a block of code  
- blackbox





class A {  
int l 3

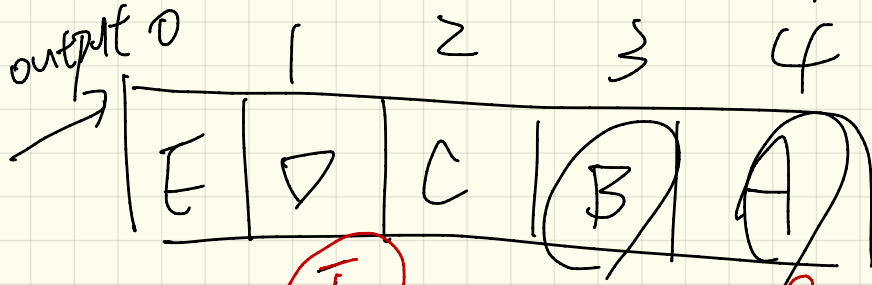
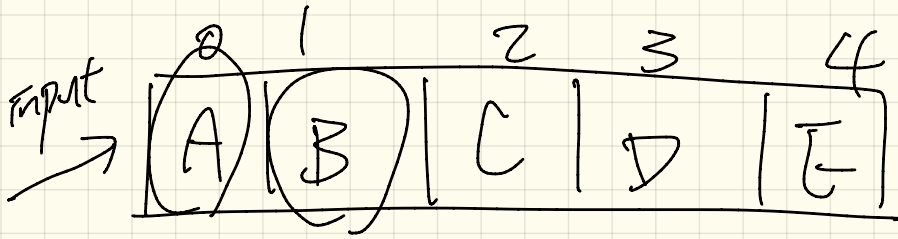
scope of j ←  
boolean am () {  
int j = 3;  
l = j \* 2; ✓  
}

void mm () {  
int k = 4;  
j l = k \* k; ✓  
k = j \* 2; ✓  
}

Tutorial Lab Test I

EXERCISES

Tuesday Sept. 26



input.length == 5

$i + ? = \text{length} - 1$   
 $? = \text{length} - i - 1$

- input [0] == output [4]
- input [1] == output [3]
- input [2] == output [2]
- input [3] == output [1]

1, 1

1, 1, 2

1, 1, 2, 3

~~2, 2~~ 4, 6, 10

1, 1, 0, 1, 2

seq: 1, 1,  $\bar{t}=2$

~~1, 1~~ 2, 3, 5, 8

a	b	c
a	c	b
b	a	c
b	c	a
c	a	b
c	b	a



3!



max ~~7~~ ~~4~~ b  
 max2 ~~2~~ 5

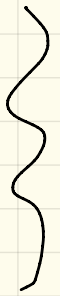
max  
 max2

Lecture 7

Thursday Sept. 28

Point P ;

default value: null



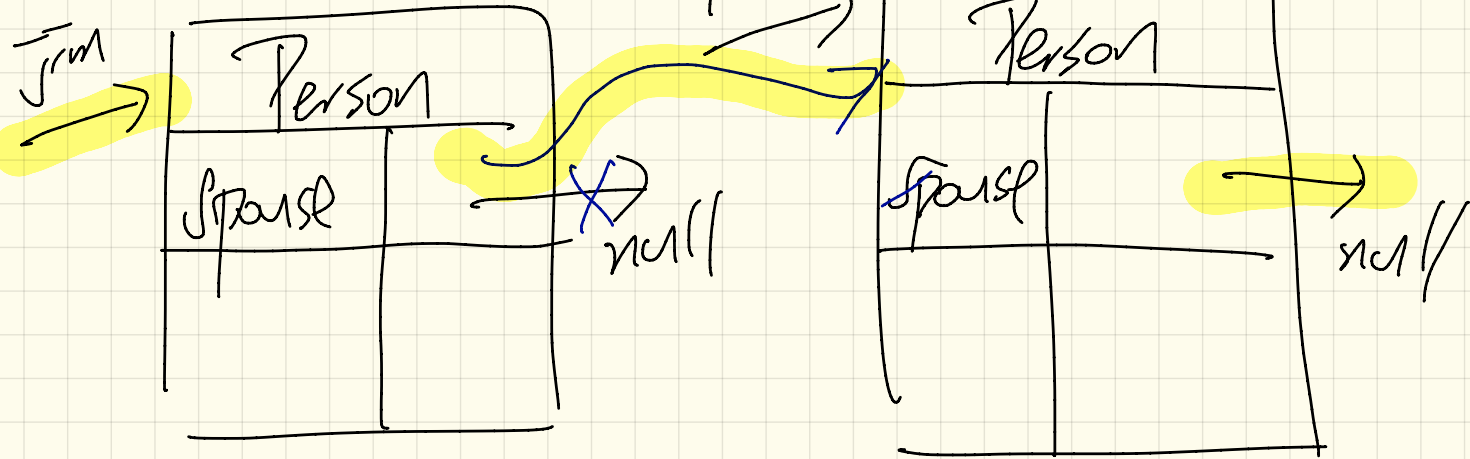
NullPointerException

P. moveUp (2);

still null

$\bar{j}m.spouse == fsa$  (true)

$\bar{j}m.spouse.spouse == \bar{j}m$  (false)



$\bar{j}m.spouse == null$   
 $fsa.spouse == null$

$\bar{j}m.married(fsa)$



```
class Person {
    String name;
    Person spouse;
```

```
if (this.spouse == null ||
    other.spouse == null) {
    Jim.marry(Lisa)
}
```

```
void marry (Person other) {
    this.spouse = other
    other.spouse = this
}

void divorce()
```

this.spouse.spouse = this.

```
class Point {
```

```
    void moveUpBy(int x2) {
```

```
        this.x + = x2;
    }
```

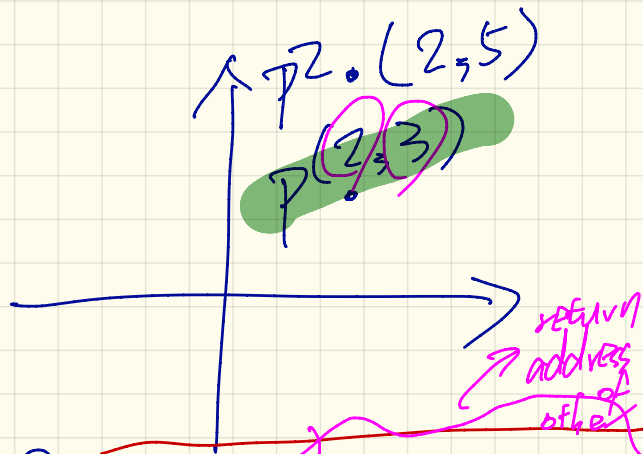
```
Point movedUpBy(int x2) {
```

```
    Point other = new Point(  
        P.this.x, P.this.y);
```

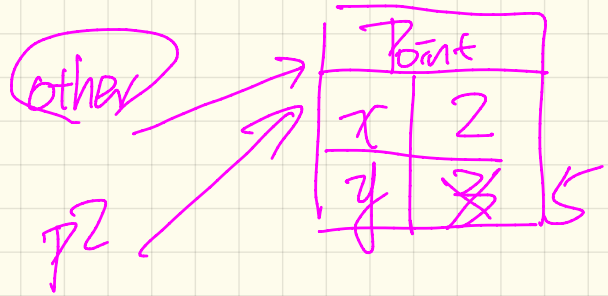
```
    other.moveUpBy(x2);
```

```
    return other;2
```

return the address of other



```
{ Point p2 = (P).movedUpBy(2); }
```



`ArrayList < Point > points =`

At runtime, 'points' stores the address of an `ArrayList` object. Every element of the `ArrayList` stores the address of some `Point` object.

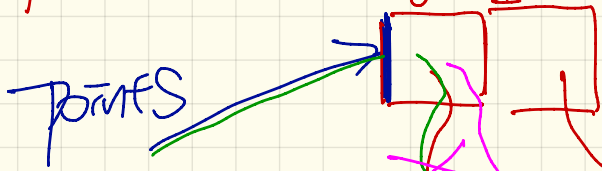
Point p1 = new Point(2, 3);

Point p2 = new Point(5, 7);

ArrayList< Point > points = new ArrayList<>();

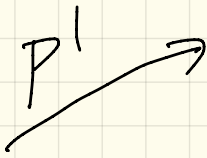
points.add(p1);

points.add(p2);



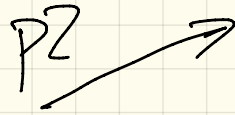
points.get(0) == p1 true  
points.get(1) == p2 true

points.set(0, p2)



points.get(0) ==  
points.get(1) true

Point	
x	2
y	3



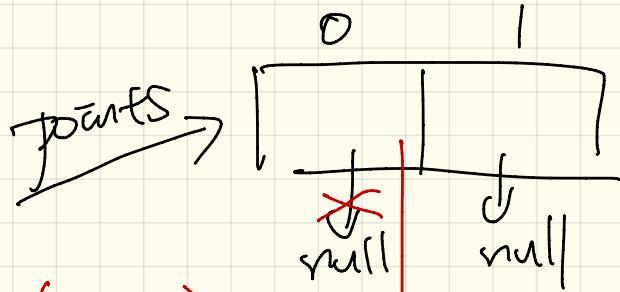
Point	
x	5
y	7

Point []

points ;

Point p = new Point  
(3, 4) ;

points = new Point [2] ;



points[0] =

new Point (3, 4) ;

Point	
x	3
y	4

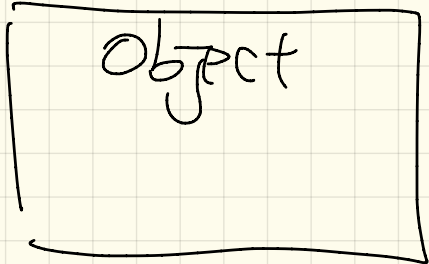
Point p1 = new Point(2, 3)

Point p2 = new Point(2, 3)

①  $p1 == p2$  false

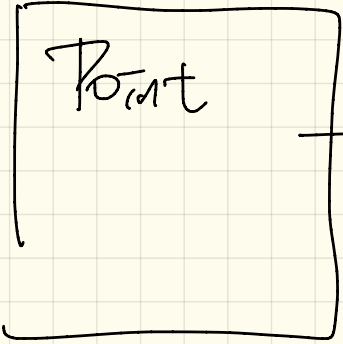
②  $[ p1.x == p2.x \ \&\& \ p1.y == p2.y ]$  true

define a helper method "equals"  
p1.equals(p2)



PARENT  
class

```
boolean equals(Object o)  
return this == o;
```



```
boolean equals(Object o)
```

```
Point equals(Point obj)  
any object
```

Point p1 = new

Point p2 = p1;

p1.equals(p2) ✓

p1.equals(null) ✗

String p3 = "(2,3)";

p1.equals(p3) ✗

Point (2,3);

Point p3 =

new Point  
(2,3);

p1.equals(p3) ✓

expanded!



Lecture 8

Tuesday Oct. 3

class Point2 {

We declare 'obj' of static type object.

boolean equals (Object obj) {

S.T. Object p3 = new Point2(...)  
D.T.

test() {  
Point2 p1 = new Point2(...);  
Point2 p2 = new Point2(...);  
} static types

dynamic type

Point2 other = (Point2) obj;

return this.x == other.x &&

this.y == other.y;

}

class Point2 {

boolean equals (Object obj) {

return [ this.x == obj.x &&  
this.y == obj.y ];

Object



Point2

double x  
double y

}

}

Fix: Since the previous 3 cases not satisfied, we've  
since 'obj' is a Point2 object

S.T. is

Object

which does not declare x and y.

```
class Point2 {
```

```
    boolean equals (Object obj) {
```

alternatively:

```
    return  
    this.x == ((Point2) obj).x &&  
    this.y == ((Point2) obj).y ;  
    }  
}
```

```
        Point2 other =  
            return y ;  
            this.x == other.x  
            &&  
            this.y == other.y ;
```

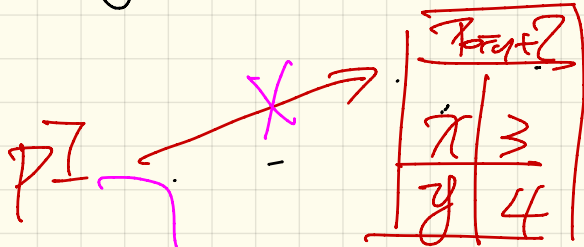
temporarily change  
the S.T. of obj  
from Object to  
Point2

once declared, can never be changed

Static type (type during declaration)

Point2 p1

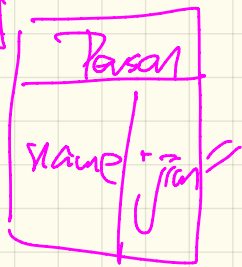
Person jcar



Dynamic type

(1) Point2 p1 = new Point2(3, 4);

(2) Object p1 = new Point2(3, 4);  
p1 = new Person("jcar");



D.T.

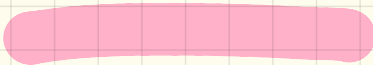
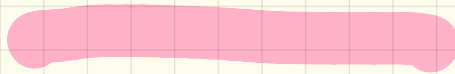
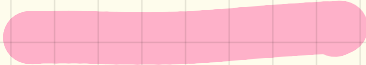
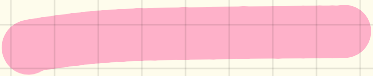

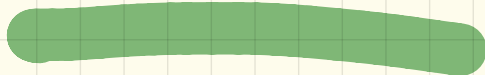
D.T. changes to Person.

Q. Swap the order: `this.getClass() != obj.getClass()`  
|| `obj == null`. ~~X~~ When `obj` is null  $\rightarrow$  Null Pointer Exception.

```
else if (obj == null || this.getClass() != obj.getClass()) {  
    return false  
}
```

Q. what if `obj` is null.

A. No.  $\therefore$  NPE.

P	Q	$P \&\& Q$	$P \parallel Q$
F	F		
F	T		
T	F		
T	T		

$P \text{ is } F$ ,

no need to  
evaluate  $Q$

$P \text{ is } T$ ,

no need to  
evaluate  $Q$ .

int  $\bar{i}$  = read from user.

$\bar{i} \in (0 \leq \bar{i} \ \&\& \ \bar{i} < a.length \ \&\& \ a[\bar{i}] > 0)$

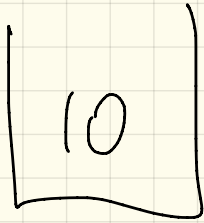
Guard to ensure  
 $\bar{i}$  is a valid index

$\bar{i} \in (r \ \&\& \ p \ \&\& \ q)$  X  
when  $\bar{i} < 0$   
r will throw  
index out of bound

$\bar{i} \in (p \ \&\& \ r \ \&\& \ q)$  X  
when  $\bar{i} \geq a.length$



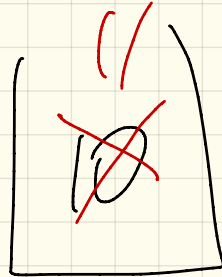
int  $\tau = 10$



$\tau$



argument



$\tau$

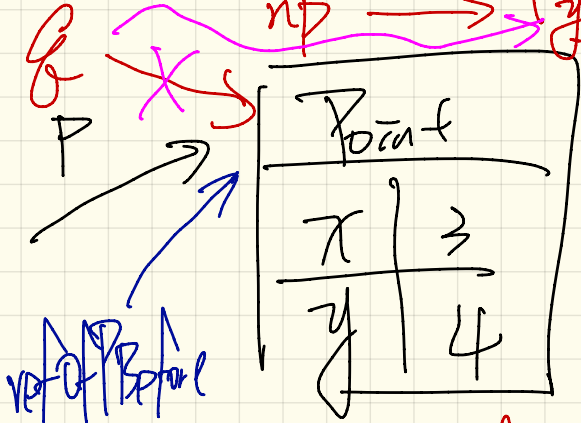


parameter

Point P = new Point(3, 4);

Point	
x	6
y	8

Point ref of P Before = P;



U. reassign Ref(P);

assert True (

P == ref of P Before);

with reassign Ref (Point [g]) {  
 Point np = new Point(6, 8);  
 } g = np;

call by ref →

Lecture 9

Thursday Oct. 5

```

class Util {
    void reassignRef (Point q) {
        Point p = new Point (6, 8);
        q = p;
    }
}

```

expecting an address of some Point object

a copy of p will replace

every occurrence of q

```

testCallByRef_1() {
    Util u = new Util();
    Point p = new Point (3, 4);
    u.reassignRef (p);
}

```

Point	
x	3
y	4

Point	
x	6
y	8

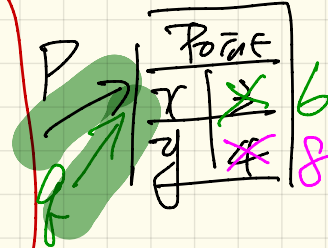


```

class Util {
    void changeViaRef(Point q) {
        q.moveH(3);
        q.moveV(4);
    }
}

```

q is replaced by a copy of p.



```

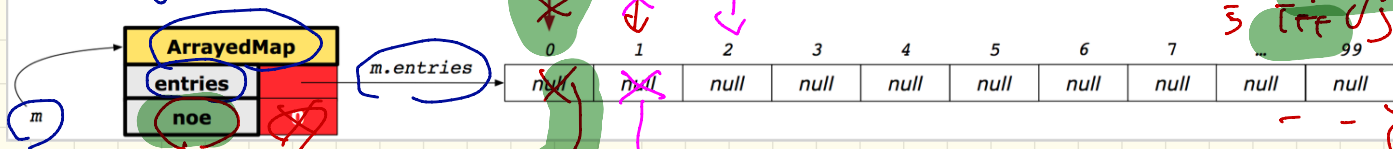
testCallByRef_2() {
    Point p = new Point(3,4);
    changeViaRef(p);
}

```

p.x = 6  
 q.y = 8  
 assertTrue(p.x == q.x)  
 p.y == q.y

Entry[] entries;

for (int i=0; i < m.entries.length; i++)



number of entries

2

Entry	
K	I
V	"D"

Entry	
K	25
V	"C"

m.entries[noe] = I

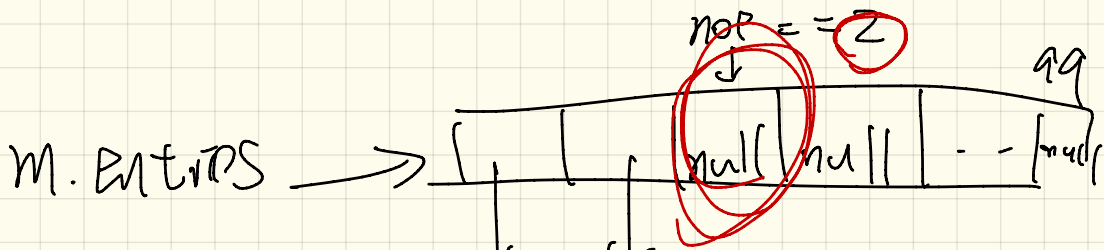
① m.put(I, "D")

② m.put(25, "C")

- Two preconditions for put
1. key already exists
  2. We already reached the max cap.

"noe" tells you

1. Number of entries in the map
2. The position of the next slot in the array to store a new entry



When  $[i == 2]$

$m.noe$   
 $m.$

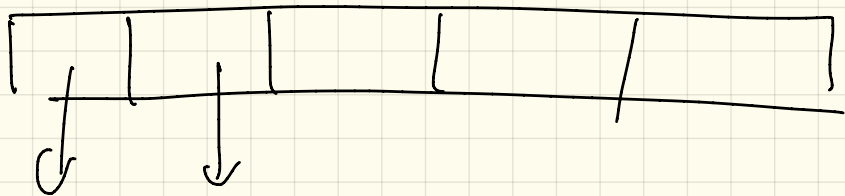
```
for (int i = 0; i < m.entries.length; i++) {
```

```
    println(m.entries[i].getKey());
```

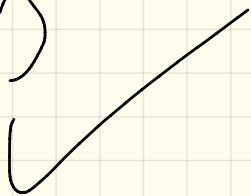
null.

```
}
```

function <sup>map</sup> vs. relation



$(1, "D")$   $(7, "D")$







Lecture 10

Tuesday Oct. 10

class Ik {

int k;  
int hashCode { return k % 11; }

boolean equals (Object other) {

return this.k.hashCode() == other.  
hashCode();

hashCode();

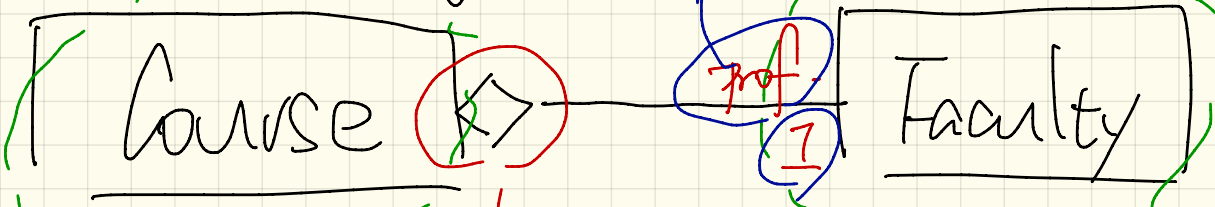
}  
Client code

Ik k1 = new Ik(3);

Ik k2 = new Ik(6);

k1.equals(k2) → true x

# UML class diagram



Container

multiplicity container  
(how many containers)

aggregation  
(sharing is allowed;  
aliasing)

Unified  
Modelling  
Language

void setProf(Faculty prof)?

this.prof = prof;

EECS3311.setProf(prof2)

EECS2030.getProf().getName().equals("Jeff")

3

course	
title	"AOP"
prof	

course

course	
title	"SD"
prof	

Faculty

EECS 3311 String

EECS2030

prof.setName("Jeff")

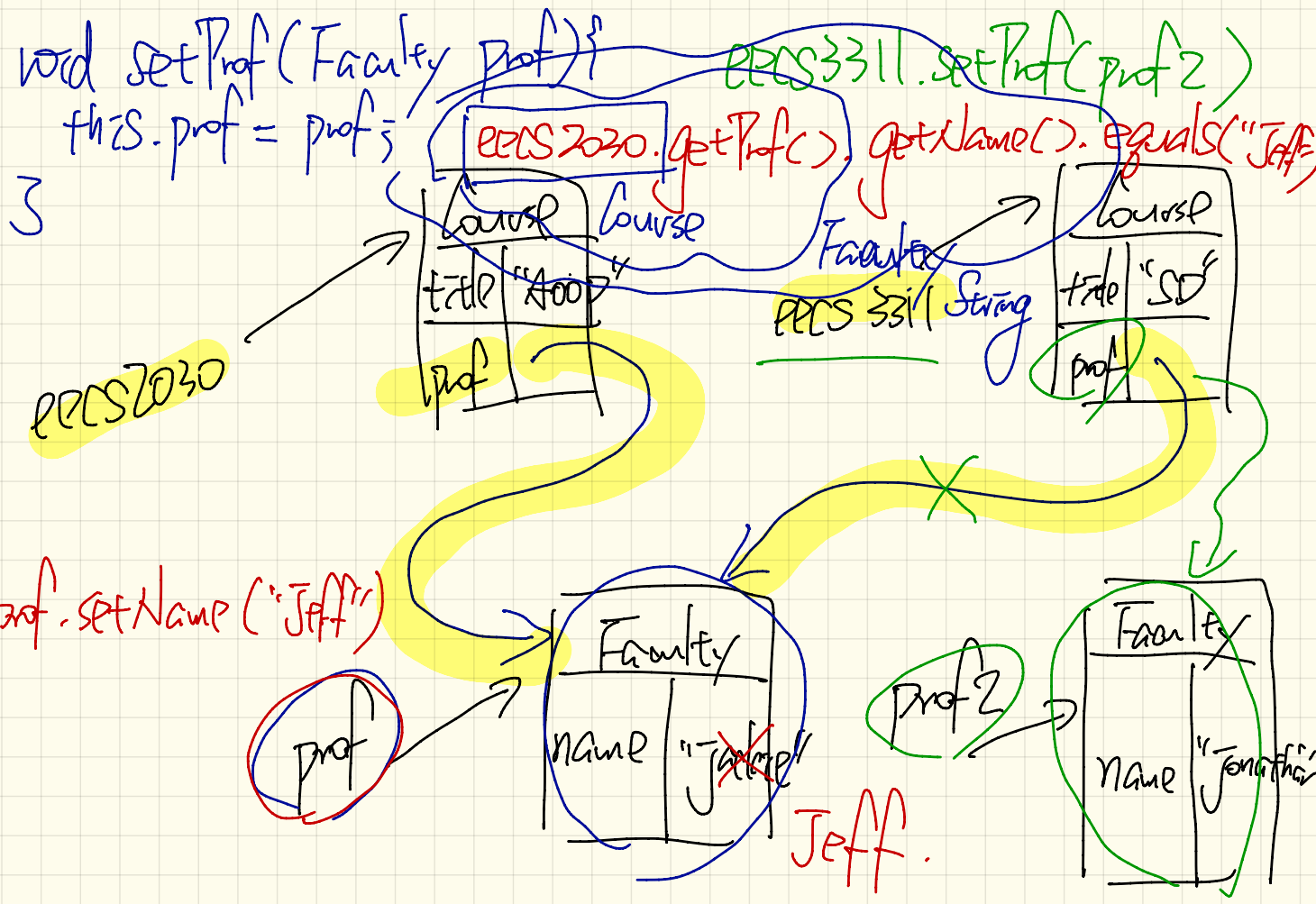
prof

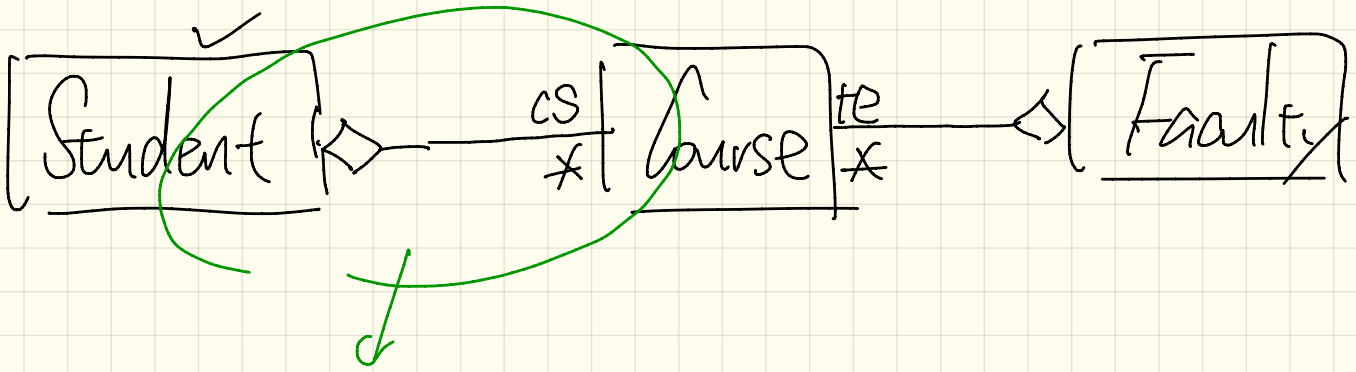
Faculty	
name	" <del>Jeff</del> "

Jeff.

prof2

Faculty	
name	"Jonathan"





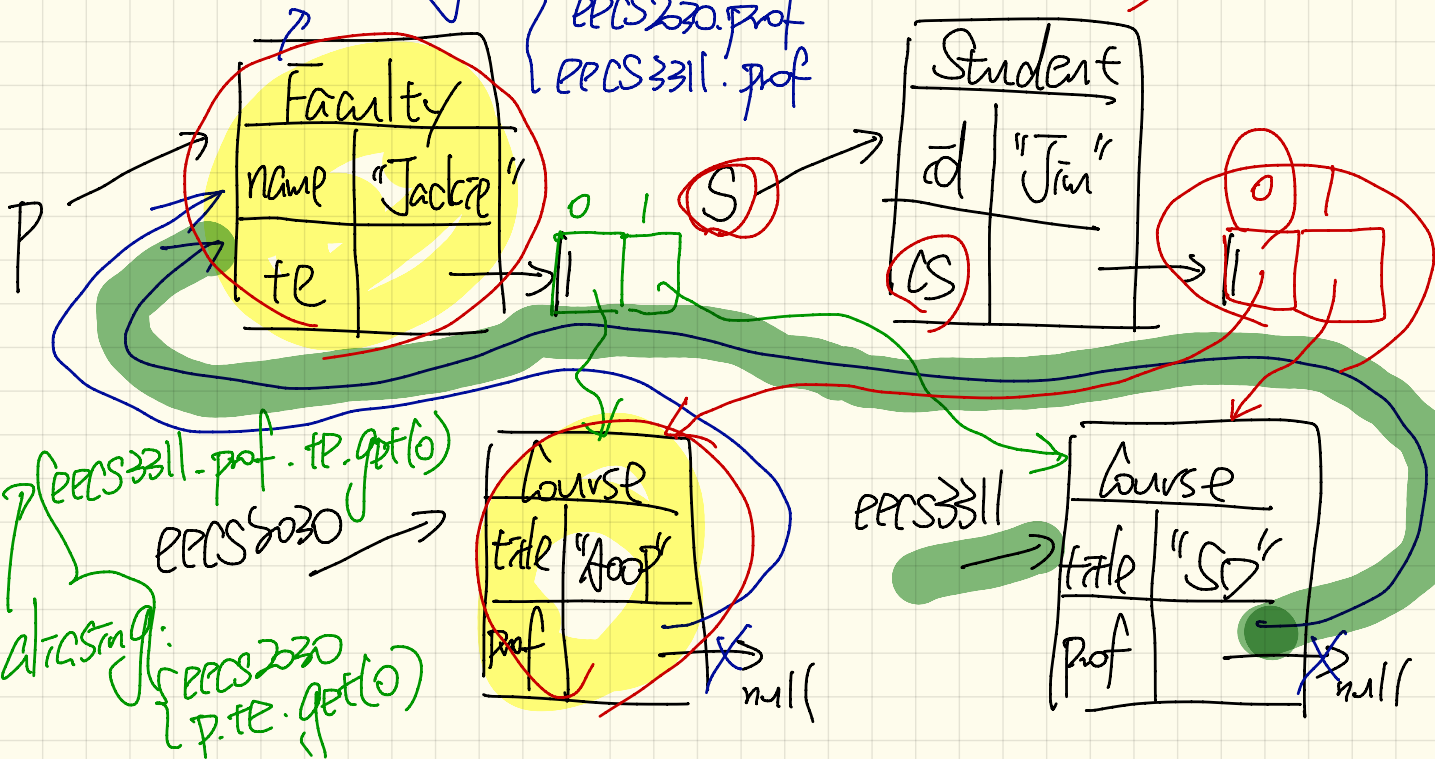
Each student has a  
collection of courses.

EPCS 2030. setProf (P)  
 EPCS 3311. setProf (P)

✓ p.addT (EPCS 2030)  
 ✓ p.addT (EPCS 3311)

s.addC (EPCS 2030)  
 s.addC (EPCS 3311)

aliasing: { P  
 EPCS2030.prof  
 EPCS3311.prof



Lecture 11

Thursday Oct. 12



```

class Directory {
    void addFile (String fn) {
        File nf = new File (fn);
        :
    }
}

```

```

} void addFile (File f) {
}
File fl = new File ("fz.txt");
dl.addFile (f);

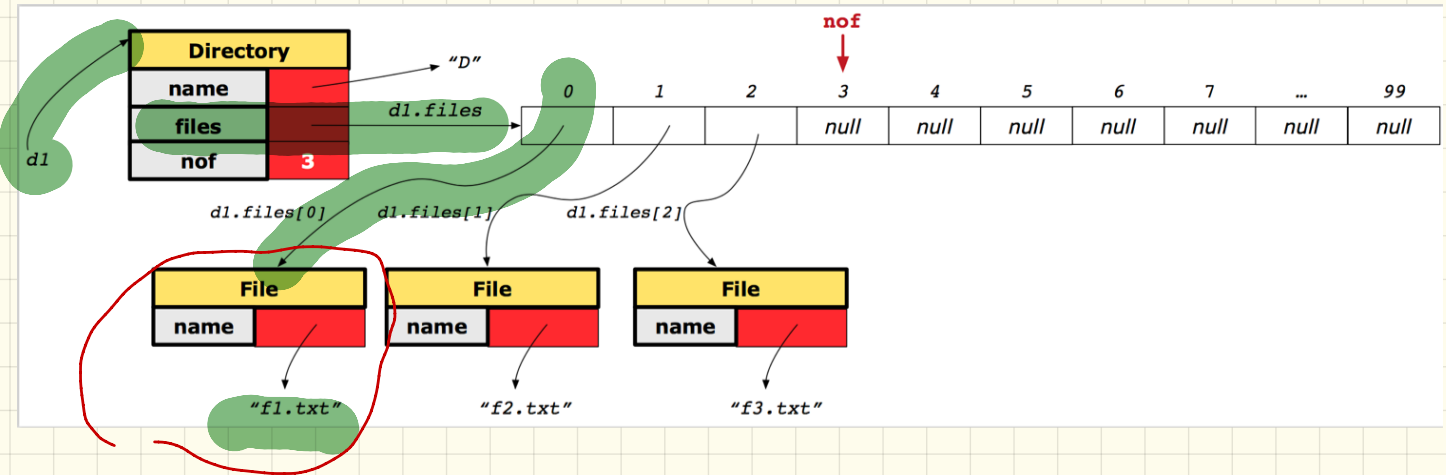
```

```

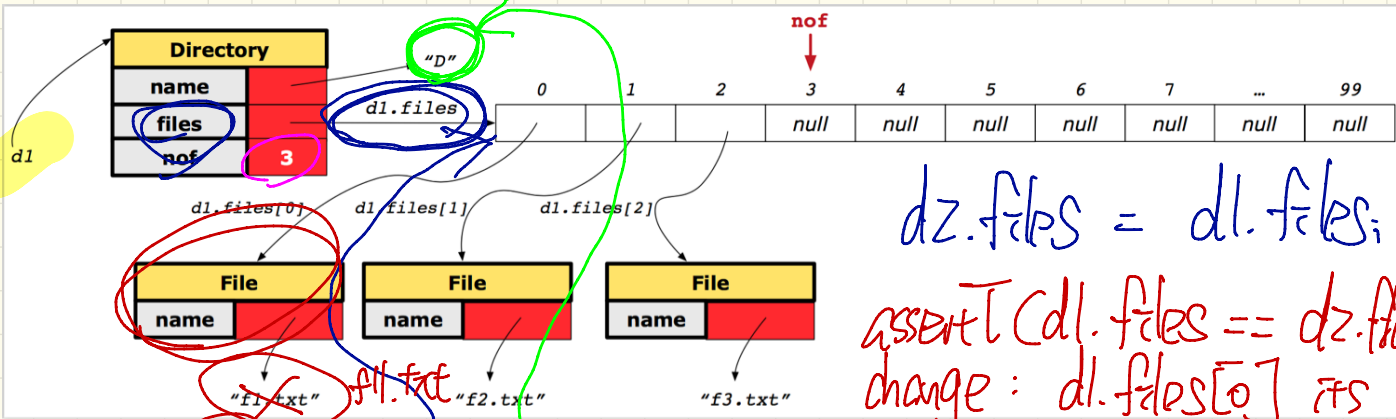
Directory dl = new Dir ("D");
dl.addFile ("fl.txt");

```

Directory dz =  
 new Directory (...);  
 Supplier  
 dz.addFile(f);  
 violation of composition (sharing) they have no reference to the client new File object created



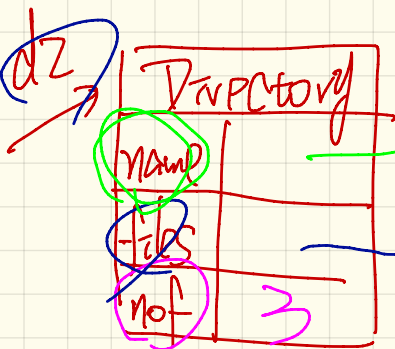
this file object  
 is only referenced  
 from d1.  
 (no sharing via  
 aliasing).



`dz.files = dl.files;`

`assert (dl.files == dz.files)`  
 change: `dl.files[0]` its name to "f1.txt"

`Directory dz = new Directory (dl);`



class Directory {  
 Directory (Directory other) {  
 printout  
 2 this.nof = other.nof;  
 ref. 3 this.files = other.files;  
 4 this.name = other.name;  
 }  
 dz  
 dl

call by ref. other is a copy of dl.

```
class CEmployee implements Comparable<...> {
```

```
    int compareTo ( CEmployee other ) {
```

```
        return this.id - other.id;
```

```
    }
```

```
} After Arrays.sort : tom, alan, mark → mark.compareTo(alan) |
```

CEmployee objects :

	ids	JAVA	<u>alan.compareTo(mark)</u> -
alan	(2)		<u>tom.compareTo(alan)</u> -

mark	(3)			
tom	1	math	<u>alan &lt; mark</u>	tom < mark
			<u>tom &lt; alan</u>	

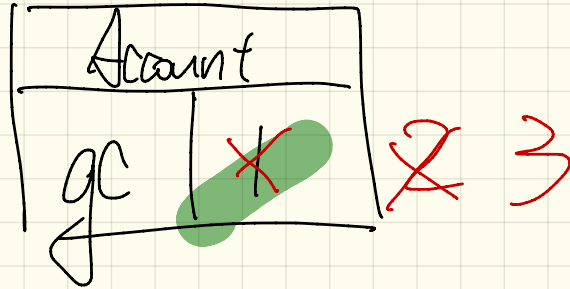
alan 5000

mark 3000

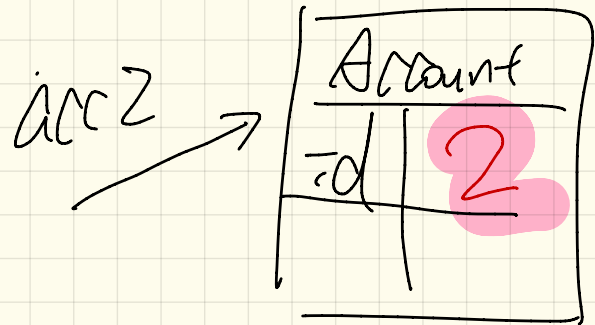
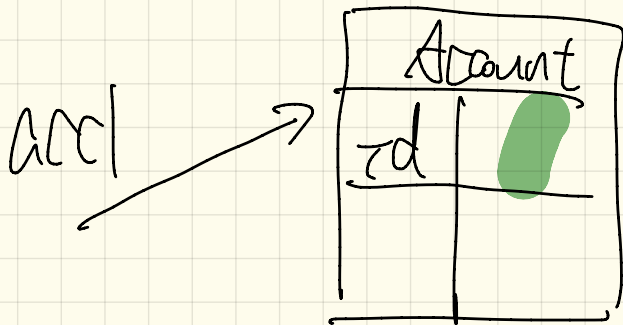
Double. compare (alan.salary, mark.salary)

+ returning this as the  
result for compare in  
Employee2 is not right

static int globalCounter



acc1 = new Account("Jim");



```
class Account {  
    String branchName;
```

```
    static int c;  
    static void m() {
```

```
        branchName
```

```
    }
```

```
Account m()
```

```
Account acc =  
    new Account("B");  
acc.branchName;
```

Lecture 12

Tuesday Oct. 17



RT = summation of RT's of all primitive operations

$$= \sum_{\bar{c}=1}^N \boxed{t_{\bar{c}}} (PM_{\bar{c}})$$

2 ops.

op1.  $t(\text{op1})$

op2.  $t(\text{op2})$

$$= C \cdot \sum_{\bar{c}=1}^N PM_{\bar{c}} = \textcircled{C} N \approx N$$

Prog A more efficient than Prog B  
 $\Rightarrow$  # p.o. of A  $\leq$  # p.o. of B

Example: Counting # of primitive operators.

```

1 findMax (int[] a, int n) {
2   currentMax = a[0];
3   for (int i = 1; i < n) {
4     if (a[i] > currentMax) {
5       currentMax = a[i]; }
6     i++ }
7   return currentMax; }
  
```

<u>i</u>	<u>i &lt; n</u>
1	T
2	T
⋮	
<u>n-1</u>	T
n	<u>F</u>

e.g. findMax ( {2, 3, 4} = 3 )

input size  
(a.length).

<hr/>	
<u>i</u>	<u>i &lt; 10</u>
1	T
2	T
⋮	
<u>9</u> <sub>10</sub>	T
10	F

~~7 · n~~ + ~~2 · n~~

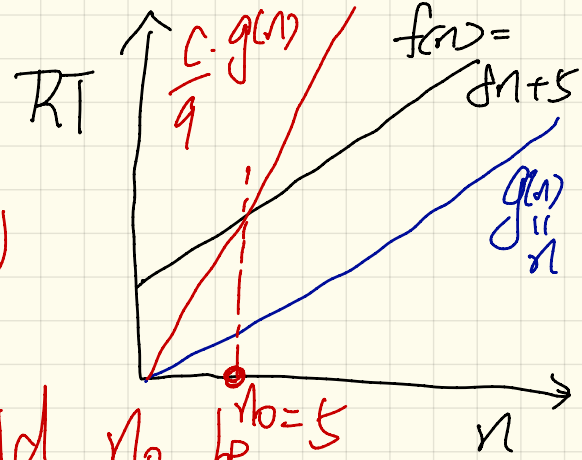
~~3(n) - 2~~

# Example: Bounding Function.

n	$8n + 5$ (code)	$9n$
1	13	9
2	21	18
3	29	27
4	37	36
5	45	45
6	53	54
...	...	...

input size

$9 \cdot n$   
 $\sim c \cdot g(n)$



What should  $n_0$  be?  
 Starting from which

$$f(n) \leq \frac{c}{9} \cdot g(n)$$

before  $n_0 = 5$ , no upper bound effect!

$f(n) = 8n + 5$  (your code)

$g(n) = n$

Show that  $f(n)$  can be bounded by  $g(n)$ :  
 choose  $c = 9$ .

$$f(n) = 8n + 5$$

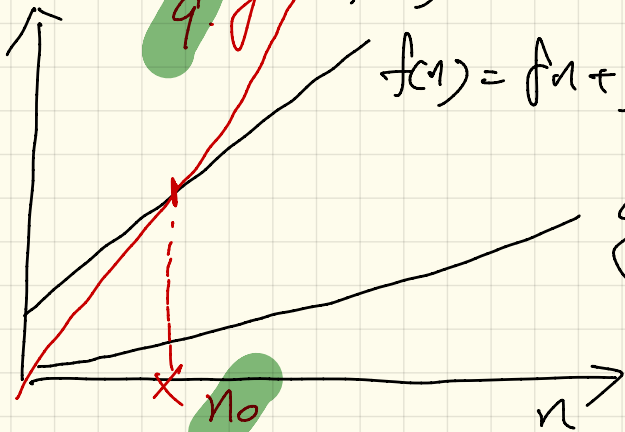
Show:  $f(n)$  is  $O(n)$

$$g(n)$$

Show that starting from  $N_0$ ,

we have  $f(n) \leq c \cdot g(n)$

RT



$$f(n) = 8n + 5$$

$$g(n) = n$$

$c$   
 $(c \cdot g(n))$

$$f(n) = a_0 \cdot n^0 + a_1 \cdot n^1 + \dots + a_d \cdot n^d$$

$$\leq a_0 \cdot n^d + a_1 \cdot n^d + \dots + a_d \cdot n^d$$

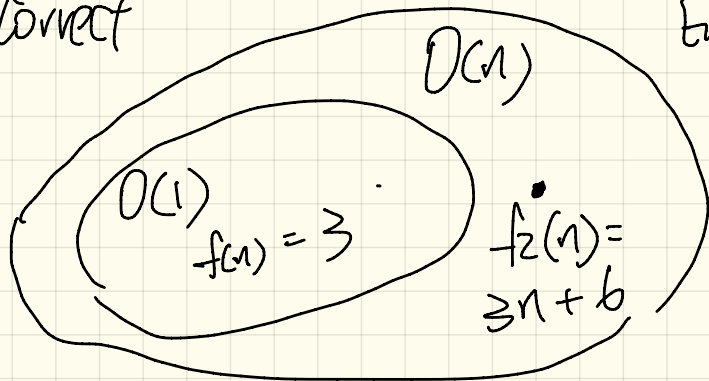
$n$  is input size

$\Rightarrow n > 0$

$\Rightarrow n^0 < n^1 < n^2 < n^3 \dots$   $\subset$

$$\leq (|a_0| + |a_1| + \dots + |a_d|) \cdot n^d$$

$\exists$  is  $O(n)$  correct  
is  $O(n)$



Every function that is  $O(n)$  is also  $O(1)$ .  
Not vice versa.

# Example: Determining Asymptotic Running Time

```
1  maxOf (int x, int y) {  
2    int max = x;  
3    if (y > x) {  
4      max = y;  
5    }  
6    return max;  
7  }
```

Lecture 13

Thursday Oct. 19

# Example: Determining Asymptotic Running Time

```
1 containsDuplicate (int[] a, int n) {  
2   for (int i = 0; i < n; ) {  
3     for (int j = 0; j < n; ) {  
4       if (i != j && a[i] == a[j]) {  $O(1)$   
5         return true; }  
6       j++; }  $\bar{i} = n-1$   
7     i++; }  $\bar{j} = 0 \dots n-1 (n)$   
8   return false; }
```

Body of loop takes  $O(1)$   $\left[ \begin{array}{l} \bar{i} = 0 \\ \bar{j} = 0 \dots n-1 \end{array} \right. (n)$

How many times?  $n^2$   $\left[ \begin{array}{l} \bar{i} = 1 \\ \bar{j} = 0 \dots n-1 \end{array} \right. (n)$

RT:  $O(1) \neq n^2 \neq O(n^2)$



# Example: Determining Asymptotic Running Time

```
1  sumMaxAndCrossProducts (int[] a, int n) {  
2  [int max = a[0];] O(1)  
3  [for(int i = 1; i < n; ) {  
4      if (a[i] > max) { max = a[i]; } } ] O(n)  
5  }  
6  [int sum = max; ] O(1)  
7  [for (int j = 0; j < n; j ++ ) {  
8      for (int k = 0; k < n; k ++ ) {  
9          sum += a[j] * a[k]; } } ] O(n2)  
10 [return sum; } ] O(1)
```

RT:  $O(n + 1 + \cancel{n^2} + 1) = O(n^2)$ .  
dominates the RT

# Example: Determining Asymptotic Running Time

```
1 triangularSum (int[] a, int n) {  
2   int sum = 0;  
3   for (int i = 0; i < n; i++) {  
4     for (int j = i; j < n; j++) {  
5       sum += a[j];  
6     }  
   return sum;  
}
```

$$\bar{i} = n-1$$

$$\bar{j} = n-1 \dots n-1 \quad (1)$$

$O(1)$

$$\bar{j} = \bar{i}$$

$$\frac{\bar{i} = 0}{\bar{j} = \frac{\bar{i}}{0} \dots n-1} \quad (n)$$

How many iterations?

$$n + (n-1) + (n-2) + \dots + 1 = \frac{n \cdot (n+1)}{2} = O(n^2)$$

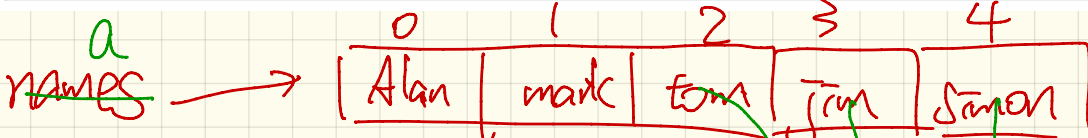
$$\frac{\bar{i} = 1}{\bar{j} = 1 \dots (n-1)} \quad (n-1)$$

# Thinking Time of Inserting into an Array [a, b]

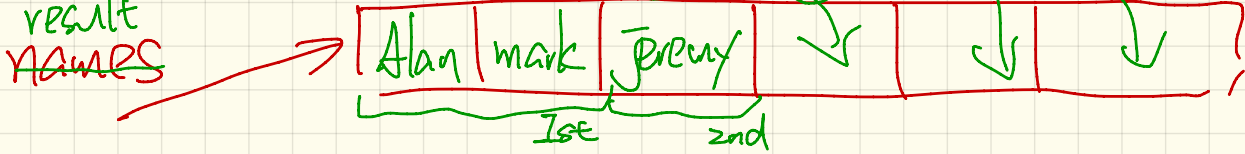
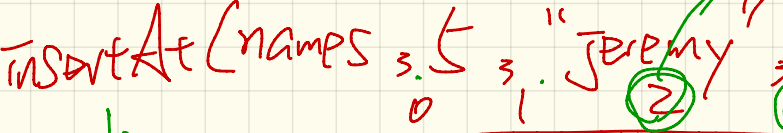
```

insertAt(String[] a, int n, String e, int i)
String[] result = new String[n + 1];
for(int j = 0; j < i; j++){ result[j] = a[j]; }
result[i] = e;
for(int j = i + 1; j < n; j++){ result[j + 1] = a[j]; }
return result;
    
```

Annotations:  $O(1)$  for array creation,  $O(i)$  for the first loop,  $O(1)$  for the assignment,  $O(n-i)$  for the second loop, and  $O(1)$  for the return statement. The total complexity is  $O(n)$ .



$result[0] = a[0]$

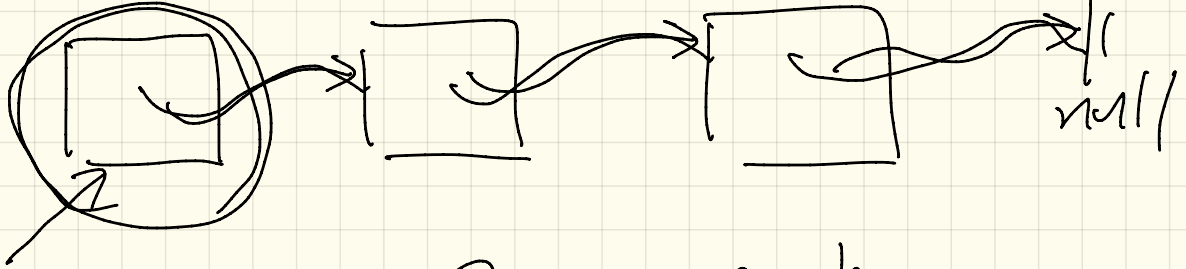


3rd:  $j = i + 1$  to  $n - 1$

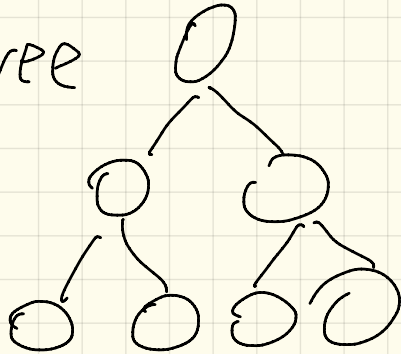
Worst case:  $i = 0$

# of iterations for 3rd step:  $n - i - 1$

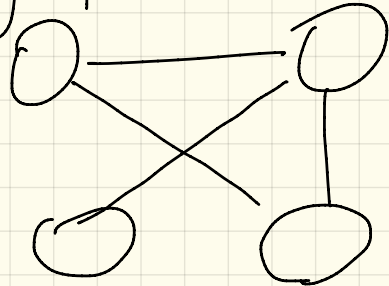
linear

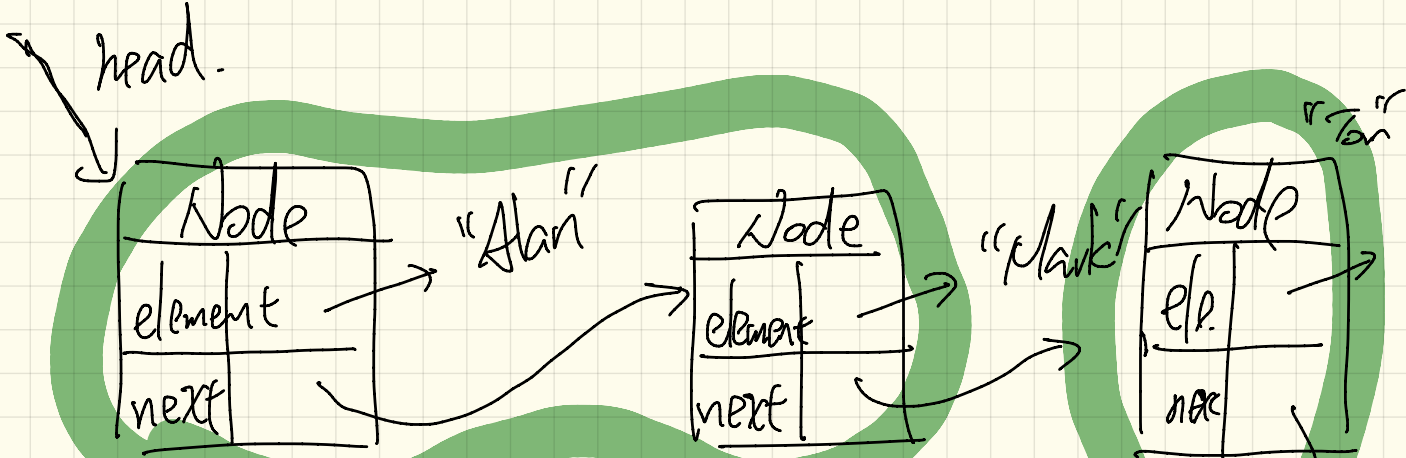


non-linear tree



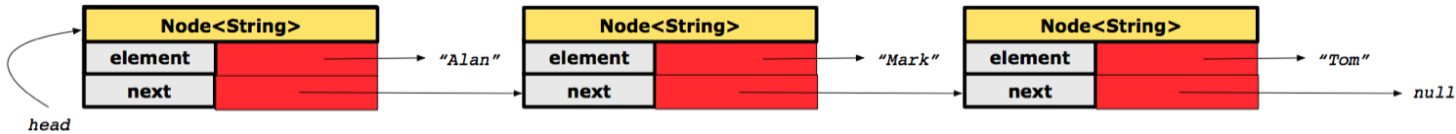
graph





a chain of two linked nodes

last nodes



```

public class Node {
    private String element;
    private Node next;
    public Node(String e, Node n) { element = e; next = n; }
    public String getElement() { return element; }
    public Node getNext() { return next; }
    public void setNext(Node n) { next = n; }
}
  
```

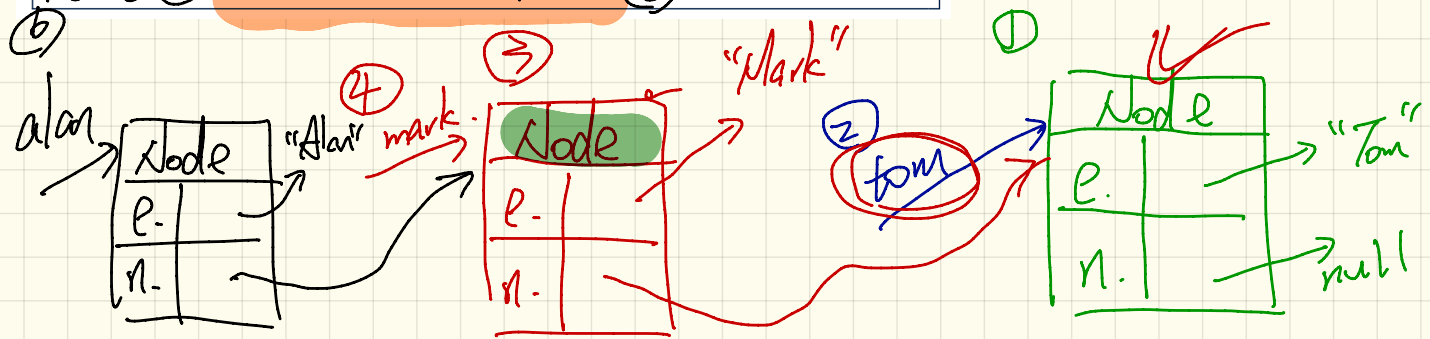
alan.next == mark

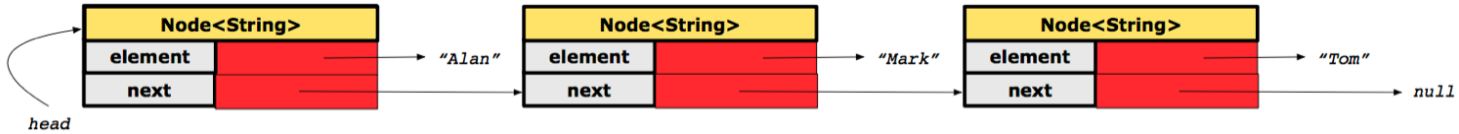
Node alan = new Node ("Alan", mark)

### Approach 1

```

Node tom = new Node("Tom", null);
Node mark = new Node("Mark", tom);
Node alan = new Node("Alan", mark);
  
```





```
public class Node {
    private String element;
    private Node next;
    public Node(String e, Node n) { element = e; next = n; }
    public String getElement() { return element; }
    public Node getNext() { return next; }
    public void setNext(Node n) { next = n; }
}
```

## Approach 2

```
Node alan = new Node("Alan", null);
Node mark = new Node("Mark", null);
Node tom = new Node("Tom", null);
alan.setNext(mark);
mark.setNext(tom);
```

Lecture 14

Tuesday Oct. 24



SLL  
empty

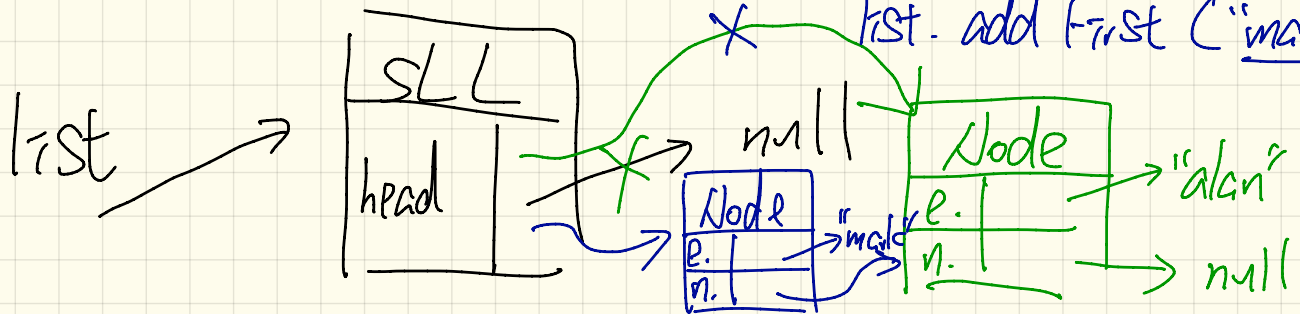
SLL list = new SLL();

↳ create an empty list

↳ [head == null]

list.addFirst("alan")

list.addFirst("mark")



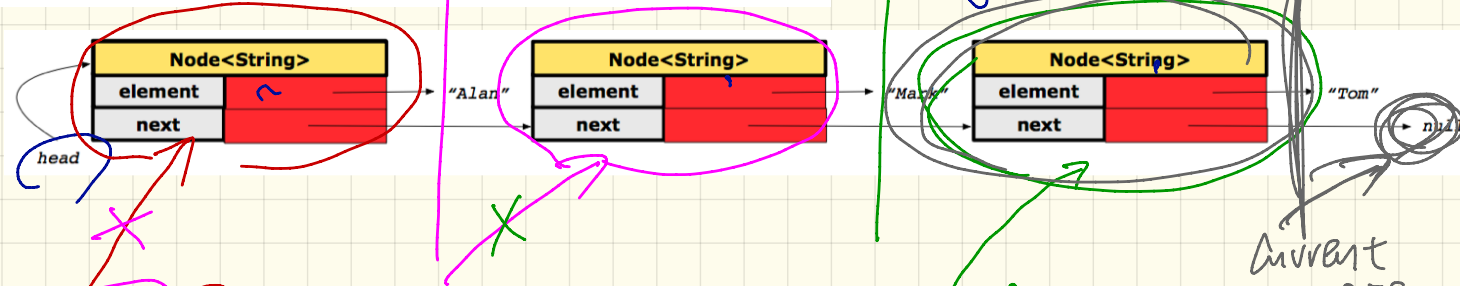
```

1  int getSize() {
2  int size = 0;
3  Node current = head; ①
4  while (current != null) {
5  /* exit when current == null */
6  current = current.getNext();
7  size ++;
8
9  return size;
10

```

As soon as current becomes null, exit from the loop.

list.getSize() returns 3



current ①

current size becomes 1

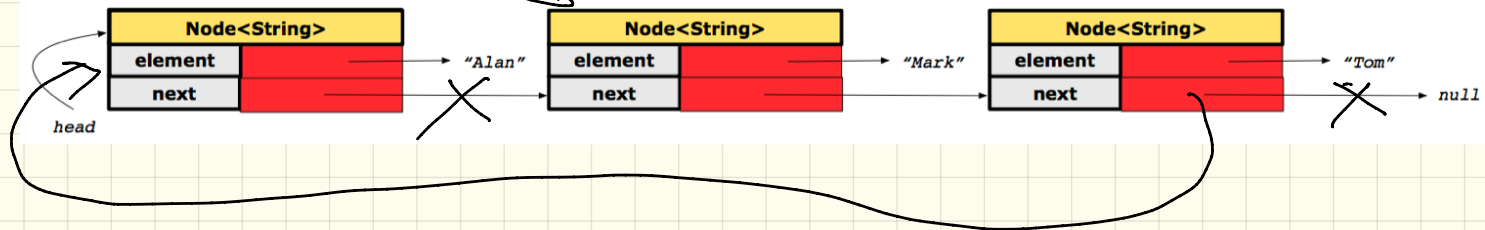
current size becomes 2 3

current = current.getNext() ②

current = current.getNext() ③

current = current.getNext()  
null

head



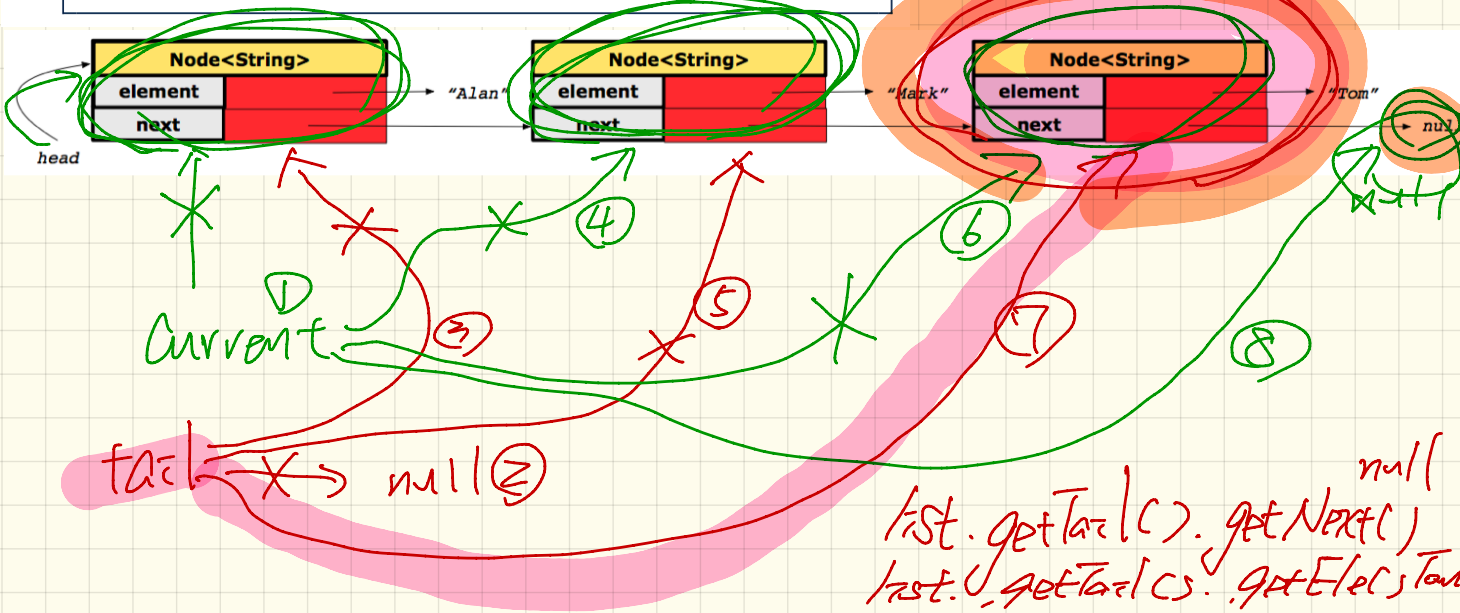
`list.shiftToLeft()`

```

1 Node getTail(Node head) {
2   Node current = head; ①
3   Node tail = null; ②
4   while (current != null) {
5     /* exit when current == null */
6     tail = current; ③ ④ ⑤ ⑥ ⑦
7     current = current.getNext(); ④ ⑥ ⑧
8   }
9   return tail;
10 }

```

list.getTail().getNext() → getNext(),  
 list.getTail().getElem() → current  
 tail



list.getTail().getNext() → null  
 list.getTail().getElem() → Tom

```
class SLL {
```

```
Node head;
```

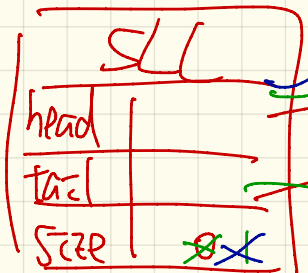
```
Node tail;
```

```
int size;
```

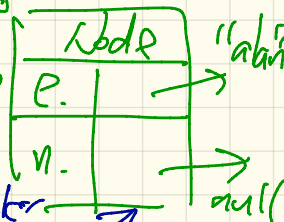
```
SLL() { head = null; tail = null; }
```

```
SLL list = new SLL();
```

list →



node



list.addFirst("alan")  
list.addFirst("mark")

"alan"

null

"mark"

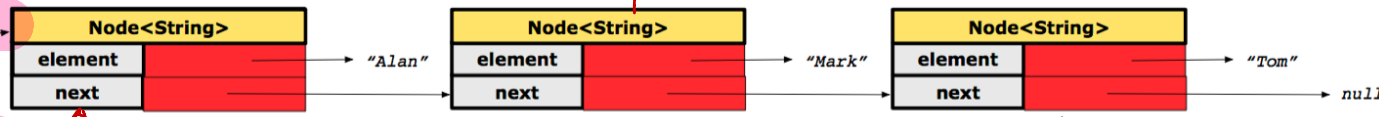
```

1  addFirst (String e) {
2  head = new Node(e, head);
3  if (size == 0) {
4      tail = head;
5  }
6  size ++;
7  }

```

Node nn = new Node(e, nn.setNext(head); null);  
 head = nn;  
 last.addFirst("Jim")

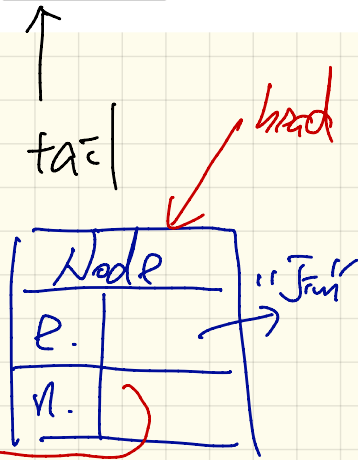
special case for adding the very first node!  
 size == 0



```

class Node {
  Node (String e, Node next) {
    this.element = e; this.next = next;
  }
}

```



```

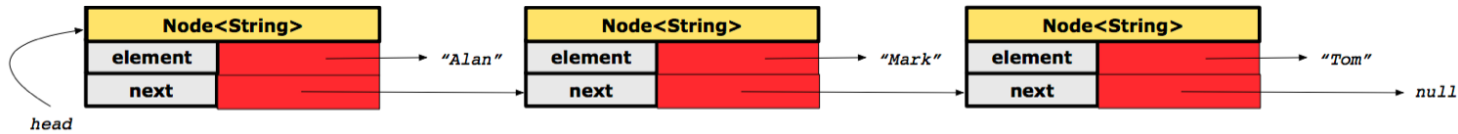
head = new Node("Jim", head)

```

```

1 Node getNodeAt (int i) {
2     if (i < 0 || i >= size) {
3         throw IllegalArgumentException("Invalid Index");
4     }
5     else {
6         int index = 0;
7         Node current = head;
8         while (index < i) { /* exit when index == i */
9             index ++;
10            /* current is set to node at index i
11             * last iteration: index incremented from i - 1 to i
12             */
13            current = current.getNext();
14        }
15        return current;
16    }
17 }

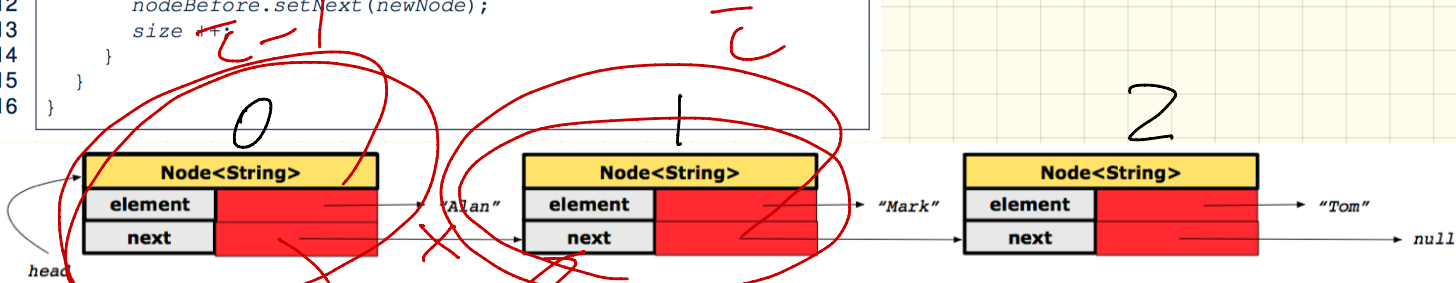
```



```

1 addAt (int i, String e) {
2     if (i < 0 || i >= size) {
3         throw IllegalArgumentException("Invalid Index.");
4     }
5     else {
6         if (i == 0) {
7             addFirst(e);
8         }
9         else {
10            Node nodeBefore = getNodeAt(i - 1);
11            newNode = new Node(e, nodeBefore.getNext());
12            nodeBefore.setNext(newNode);
13            size++;
14        }
15    }
16 }

```



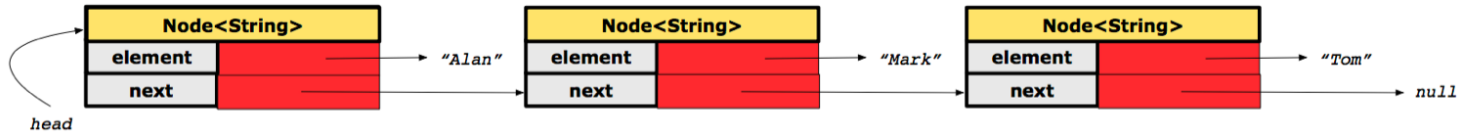
list.addAt(1, "Jean")



```

1  removeLast () {
2      if (size == 0) {
3          System.err.println("Empty List.");
4      }
5      else if (size == 1) {
6          removeFirst();
7      }
8      else {
9          Node secondLastNode = getNodeAt(size - 2);
10         secondLastNode.setNext(null);
11         tail = secondLastNode;
12         size --;
13     }
14 }

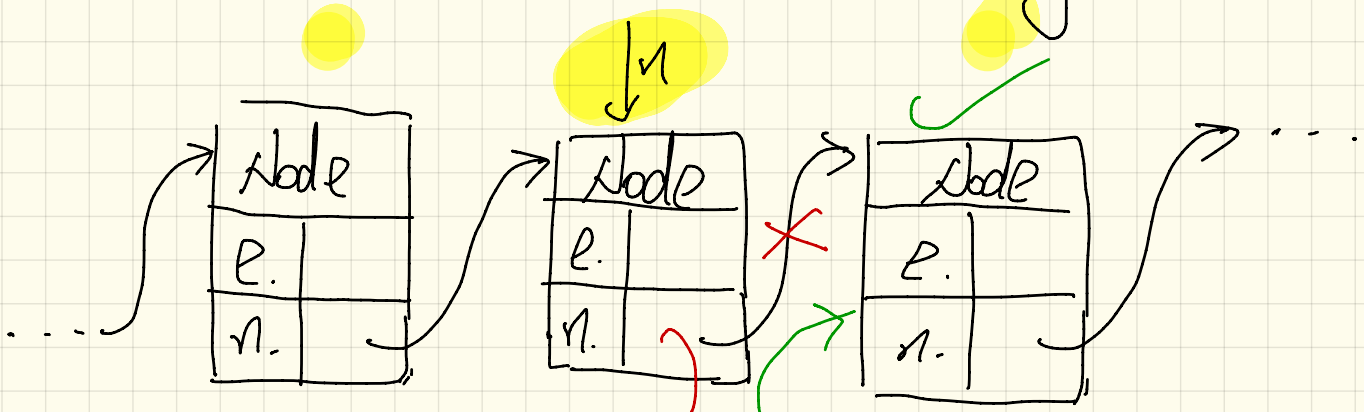
```



Lecture 15

Tuesday Oct. 31

void insertAfter (Node n, String e)

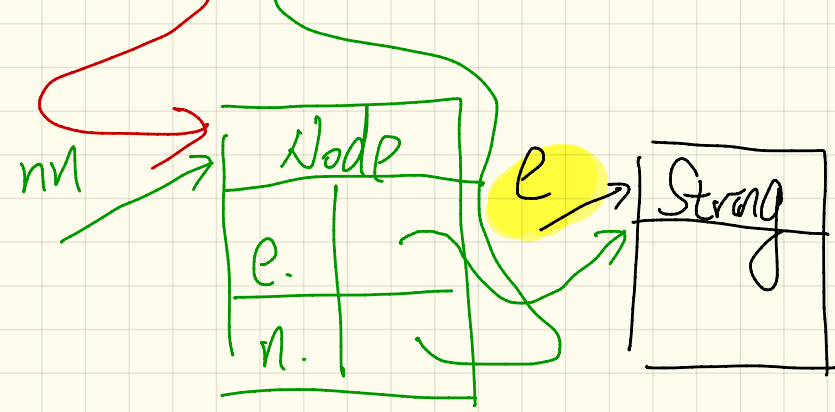


nn. element = e

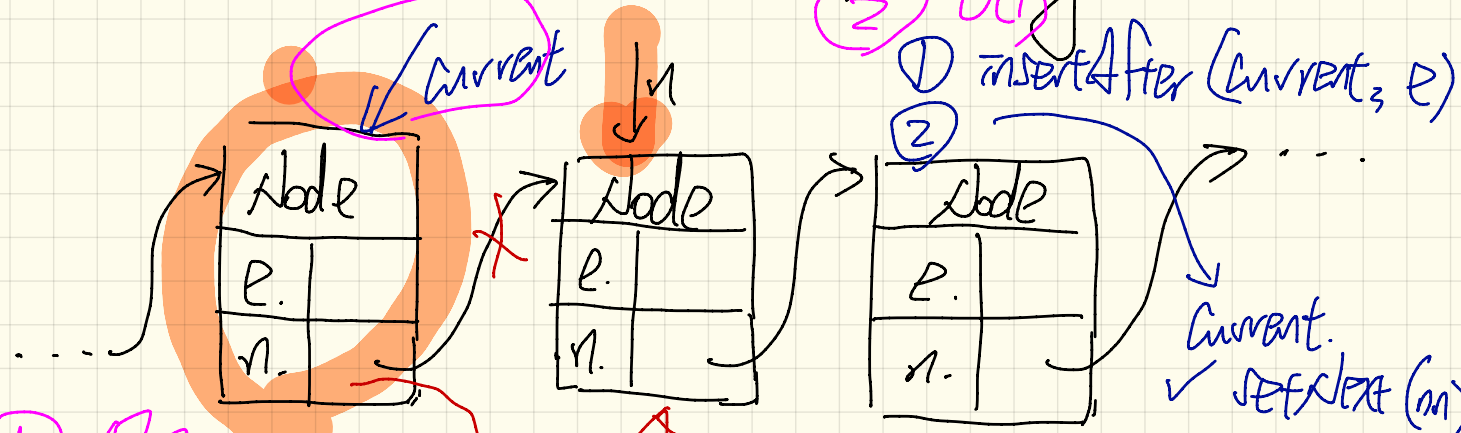
nn. setNext (n.next)

n. setNext (nn)

size ++ ;

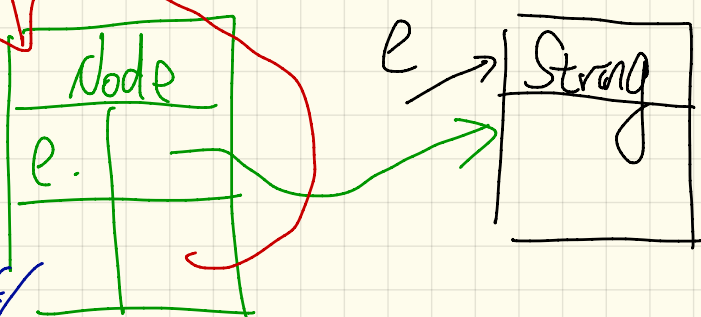


void insertBefore (Node n, String e)



①  $O(n)$

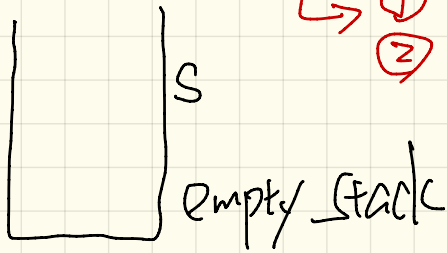
```
Node current = head;
while (current.next != n) {
    current = current.next;
}
→ current.next == n
```



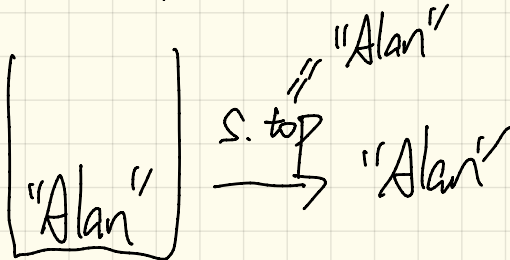
# Stack Operations

push, pop, top, size

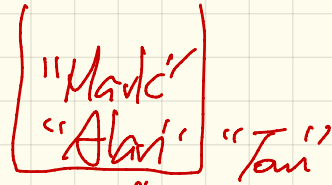
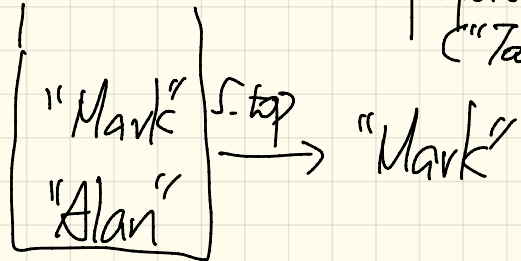
- ① return the current top
- ② remove the top



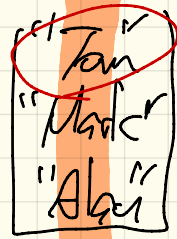
↓ s.push("Alan")



s.push("Mark")



s.pop()



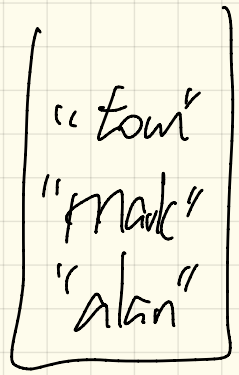
s.push("Jan")

empty  
✓ Stack S

S.push("alan")

S.push("mark")

S.push("tom")



S.pop()

S.pop()

S.pop()

S.size()

"tom" "mark" "alan"

reverse order for pushing.

# Implement a Stack using Array

$t$ : index of the top.

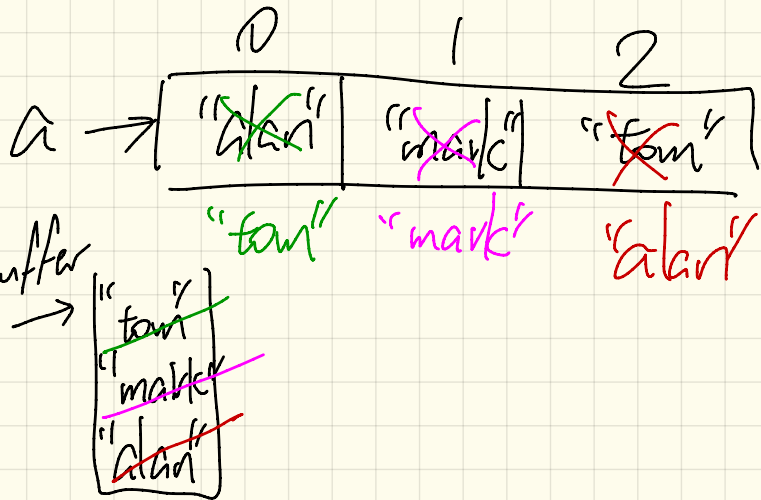


$t = -1$   
 $t = 0$   
 $t = 1$

(empty stack)

$s.pop()$  →  $data[t] = null$   
 $t--$  retain of  $s$   
 $s.top()$  →  $data[t]$

$s.push("alan")$  →  $s.top()$  →  $s.push("mark")$   
↳  $t++$   
 $data[t] = "alan"$       ↳  $data[t]$       ↳  $t++$   
 $data[t] = "mark"$

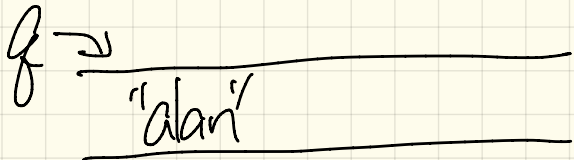
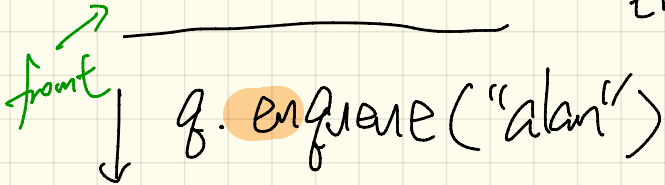


```
int i=0;
while (!buffer.isEmpty()) {
    a[i] = buffer.pop();
    i++;
}
```

```
for (int i=0; i < a.length; i++) {
    0
    a[i] = buffer.pop();
    1
    2
    "marc"
    "alan"
```

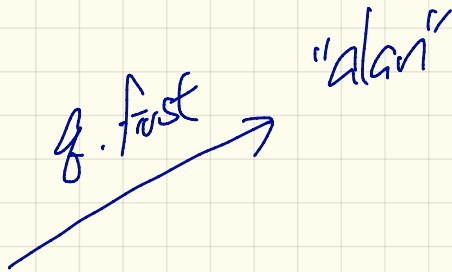
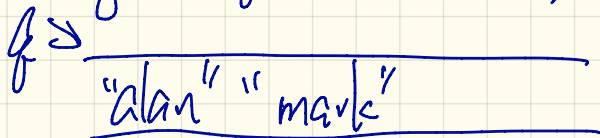


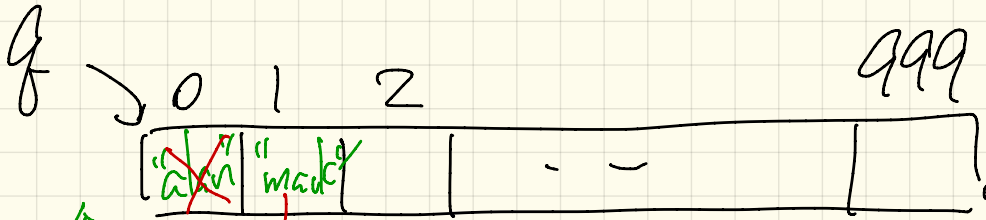
# FIFO Queue



↓ q.front "alan"

q.enqueue("mark")





↑  
~~\*~~  
 $r == -1$   
 rear  
 of queue

q.first() → q[0]

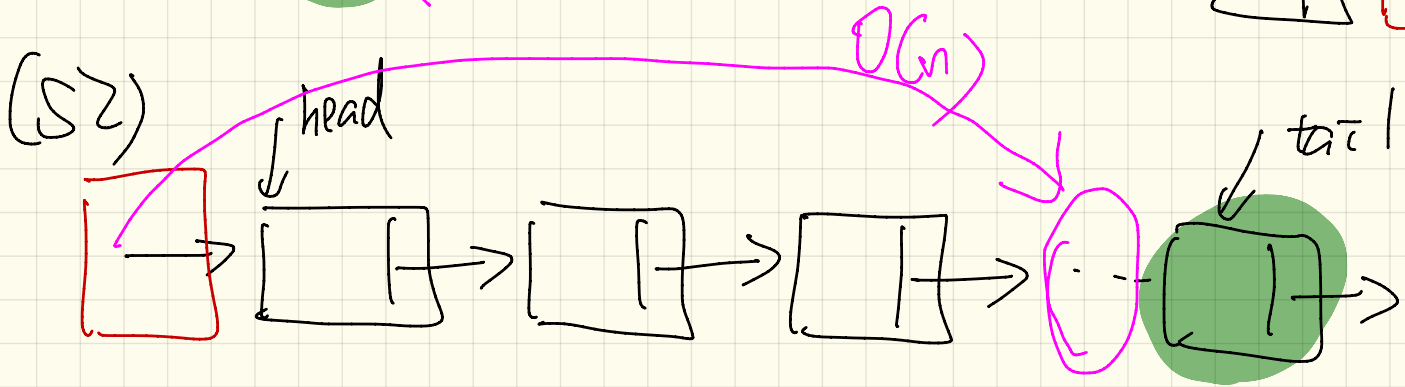
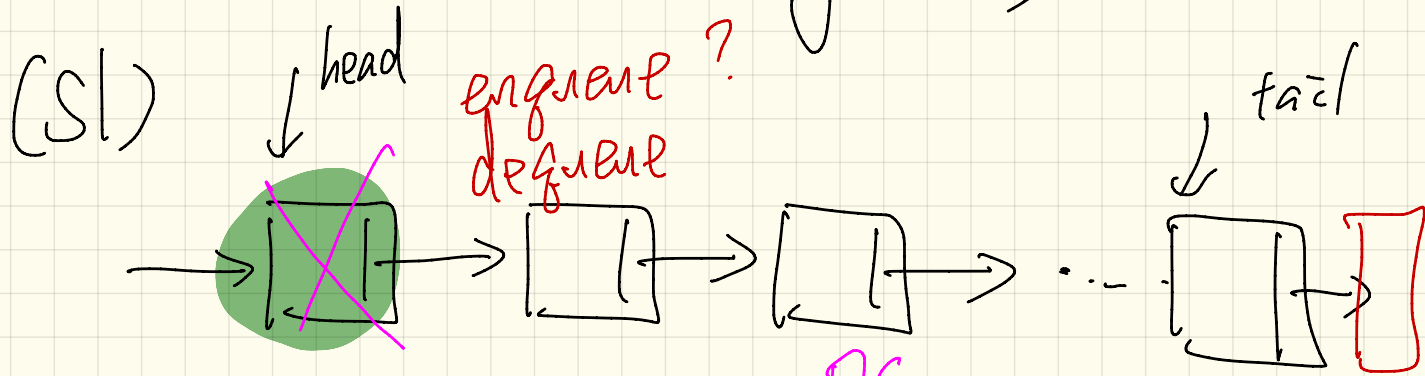
q.dequeue() → O(n)  
 shift

q.enqueue("alan") → q.enqueue("mark")

↳  $r++$ ;  
 $q[r] = \text{"alan"}$

↳  $r++$ ;  
 $q[r] = \text{"mark"}$

# Implement the Q using SLL

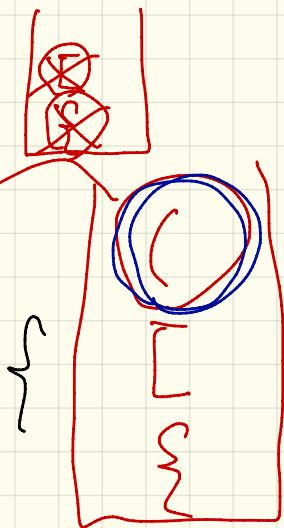


{ [ ( ) ] }

{ [ ] } ( )

open = "{ [ ("  
close = ")} ] )"

{ [ ( ) ] } ✓



{ [ ( ) ] }

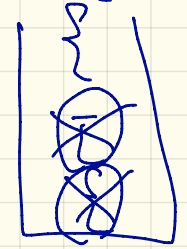
X

{ [ ] } ( )

{ [ ] } ( ) X

{ [ ( ) ] } )

closing



{ [ ] } { X

Lecture 16  
Thursday Nov. 2

$$\underbrace{P(n)} = \frac{(1+n)n}{2}$$

Sum of  
the first  
 $n$  integers  
down!

Base Cases

$P(1)$   $P(2)$

Recursive/Inductive Cases

Assume:  $P(n-1) = \frac{(1+(n-1))(n-1)}{2}$

Prove:  $P(n)$  (derive)

Problem :  $n!$       $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$   
 size of problem      $4!$   
 $7! = 7 \times 6!$

$$n! = \begin{cases} 1 & n=0 \\ n \cdot \underbrace{(n-1) \cdot (n-2) \cdots 1}_{(n-1)!} & n > 0 \end{cases}$$

① Base Cases

$(n-1)!$

$$n! = n \times (n-1)!$$

$0! = 1$       $1! = 1$

② Recursive Cases

Assume we have

$(n-1)!$

Solution to a strictly smaller problem.

Define:

$$\underbrace{n!}_{\text{original problem}} = \underbrace{(n-1)!}_{\text{sub-problem}} \times n$$

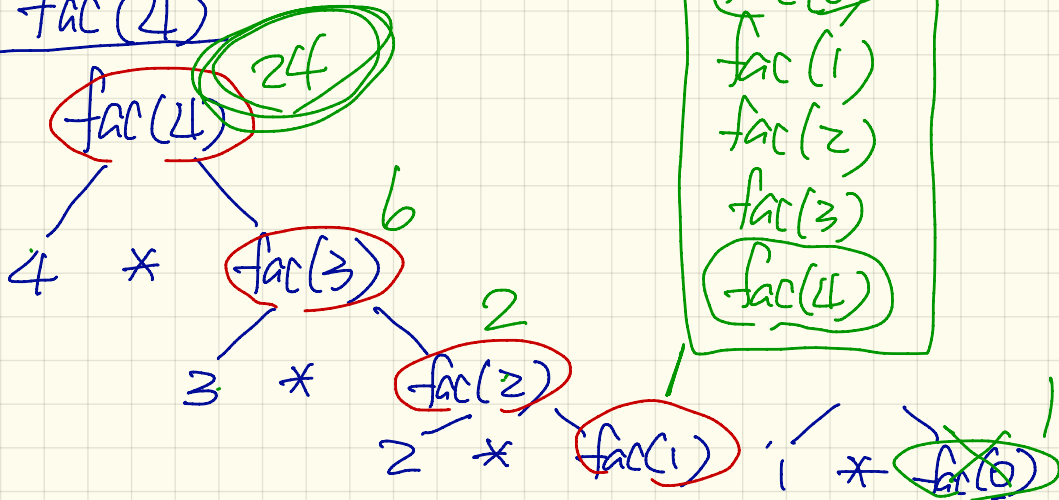
sub-problem

```

int factorial (int n) {
    int result;
    if (n == 0) { /* base case */ result = 1; }
    else { /* recursive case */
        result = n * factorial (n - 1);
    }
    return result;
}

```

Tracing  $fac(4)$





```

int factorial (int n) {
    int result;
    if (n == 0) { /* base case */ result = 1; }
    else { /* recursive case */
        result = n * factorial (n - 1);
    }
    return result;
}

```

8. fac(0)  
returns 1  
pop()

9. fac(1)  
returns 1  
pop()

10. fac(2)  
returns 2

11. fac(3)  
returns 6 pop()

Tracing fac(3)

1. Activate fac(3),  
push ( fac(3) )

2. Execute fac(3) 2  
↳ 3 \* fac(2)

3. Activate fac(2),  
push ( fac(2) )

4. Execute fac(2)  
↳ 2 \* fac(1)

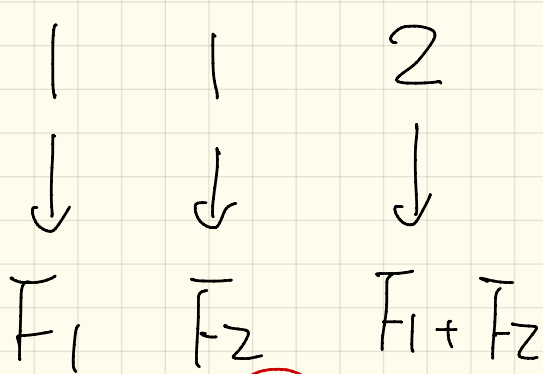
5. Activate fac(1)  
↳ push ( fac(1) )

6. Execute fac(1)  
↳ 1 \* fac(0)

7. Activate fac(0)  
push ( fac(0) )

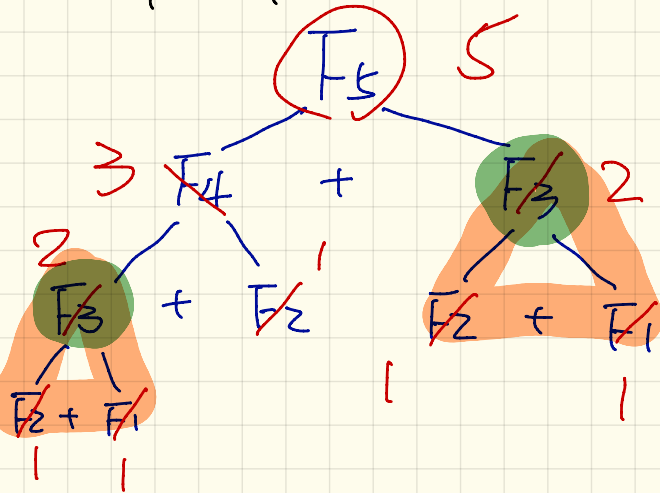


# Fib. Seq.



$$F_n = \begin{cases} 1 & n=1 \\ 1 & n=2 \\ \boxed{F_{n-1}} + \boxed{F_{n-2}} & n > 2 \end{cases}$$

solution to a strictly smaller problem  
 solution to smaller problem



```
int factorial (int n) {
    int result;
    if (n == 0) { /* base case */ result = 1; }
    else { /* recursive case */
        result = factorial factorial (n-1 n);
    }
    return result;
}
```

fac(5)

|  
fac(5)

|  
fac(5)

|  
|  
|  
|  
|  
|

```

int fib (int n) {
    int result;
    if (n == 1) { /* base case */ result = 1; }
    else if (n == 2) { /* base case */ result = 1; }
    else { /* recursive case */
        result = fib (n - 1) + fib (n - 2);
    }
    return result;
}

```

6. Execute fib(1)  
return 1  
pop

7. Execute fib(2)  
return  
1 + 1 = 2  
pop 1

fib(3)

1. Activate fib(3)

↳ push (fib(3))

3. Activate fib(2)  
push

4. Execute fib(2)

↳ return 1

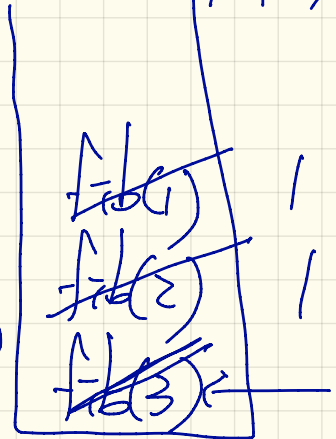
↳ pop

5. Activate fib(1)

↳ push

2. Execute top of stack fib(3)

↳ fib(2) + fib(1)



```

public class StringTester {
    public static void main(String[] args) {
        String s = "abcd";
        System.out.println(s.isEmpty()); /* false */
        /* Characters in index range [0, 0) */
        String t0 = s.substring(0, 0);
        System.out.println(t0); /* "" */
        /* Characters in index range [0, 4) */
        String t1 = s.substring(0, 4);
        System.out.println(t1); /* "abcd" */
        /* Characters in index range [1, 3) */
        String t2 = s.substring(1, 3);
        System.out.println(t2); /* "bc" */
        String t3 = s.substring(0, 2) + s.substring(2, 4);
        System.out.println(s.equals(t3)); /* true */
        for(int i = 0; i < s.length(); i++) {
            System.out.print(s.charAt(i));
        }
        System.out.println();
    }
}

```

$s.substring(i, j)$

$[i, j)$

0, -1

String s = "York";

String s2 = s.substring(0, 3)

"-York"

String  $S = \text{"..."}'$   
 $0 \leq i < S.length() - 1$

$S.substring(0, i) + S.substring(i, S.length())$   ~~$\times$~~

$S = \text{"love"}$   $S.length() = 4$

$S.substring(0, 2) + S.substring(2, 4)$   
 $\begin{matrix} 0 & 1 & & 2 & 3 & 4 \\ \text{"lo"} & & & \text{"rk"} & & \end{matrix}$

✓ ra ec ar ✓

Base Cases:

"" is P  
" " is P

ra ec ar  
x x

Recursive Cases:

reverse  
input: a b c d

a b c d  
d c b a

reverse(b c d) + a

output: d c b a

reverse(" ") " "

reverse(" \_ ") " \_ "



Lecture 17

Tuesday Nov. 7

Problem	Sub-Problems	Solution
fac(n)	fac(n-1)	$n \times \text{fac}(n-1)$
fib(n)	$\frac{\text{fib}(n-1)}{\text{fib}(n-2)}$	$\text{fib}(n-1) + \text{fib}(n-2)$
isP( <del>C1</del> , C2)	isP(S)	$C_1 == C_2$ <del>isP(S)</del> $C_1 == C_2$ $\text{isP}(S)$
reverse(C, S)	reverse(S)	$\text{reverse}(S) + C_1$ <div style="border-left: 1px solid green; padding-left: 10px; margin-left: 20px;"> <math>\downarrow</math> if <math>C_1 == C_2</math>  <math>0 + \text{occOf}(S, 'c')</math>  else  <math>0 +</math> </div>
occOf(C1, C2)	<u>occOf(S, C)</u>	<div style="border: 1px solid green; border-radius: 50%; padding: 10px;"> <math>\text{occOf}('a' \text{aab}', 'a') = 1 + \text{occOf}('aab')</math>  <math>\text{occOf}('baab', 'a') = 0 + \text{occOf}('aab')</math> </div>

$$\forall x : \mathbb{int} \mid x \in \{1, 2, 3\} \cdot x > 2 \quad \text{F}$$

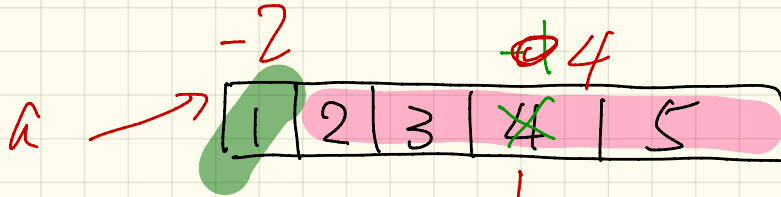
$$\exists x : \mathbb{int} \mid x \in \{1, 2, 3\} \cdot x > 2 \quad \text{T}$$

(1)

$$\forall x \mid x \in \emptyset \cdot P(x) \quad \text{T} \quad \begin{array}{l} \text{! no counter-ex.} \\ e \text{ s.t. } e \in \emptyset \\ \uparrow \\ \neg P(e) \end{array}$$

$$\exists x \mid x \in \emptyset \cdot P(x) \quad \text{F} \quad \begin{array}{l} \text{! no witness} \\ e \text{ s.t. } e \in \emptyset \\ \uparrow \\ P(e) \end{array}$$

isAllPos (int[] a)



isAllPositive ({2, 3, 4, 5})

↪ ~~True~~ ~~True~~  
~~True~~ ~~True~~

$a[0] > 0$   
~~True~~

isAllPos ({2, 3, ~~4~~, 5})  
~~True~~

isAllPos({2, 3, -1, 3})

2 > 0  
T

&&

isAllPos({3, -1, 3})

0	1	2	3
2	3	-1	3

a ↑  
from sa1  
↑  
from sa2  
↑  
to sa3

from to sa3

3 > 0  
T

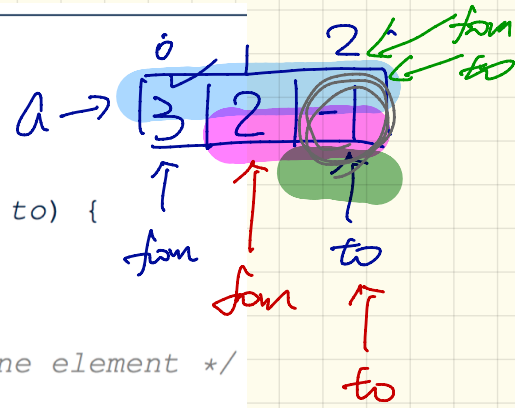
&&

isAllPos({-1, 3})

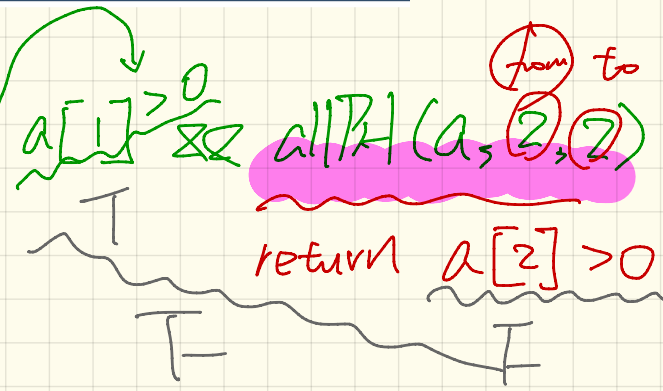
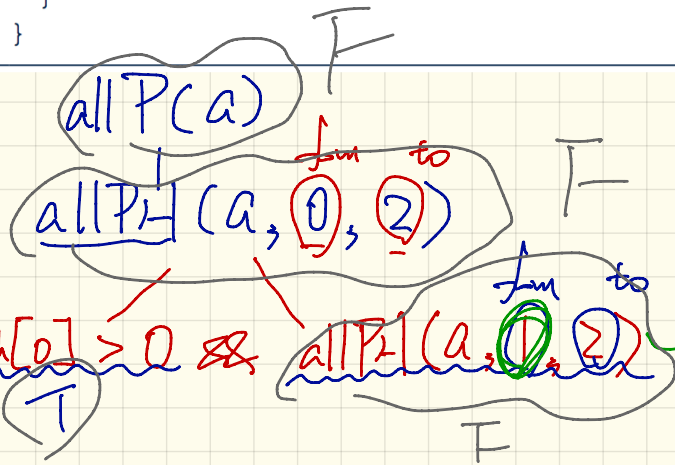
-1 > 0  
F

isAllPos({3})

```
boolean allPositive(int[] a) {
    return allPositiveHelper(a, 0, a.length - 1);
}
```



```
boolean allPositiveHelper(int[] a, int from, int to) {
    if (from > to) { /* base case 1: empty range */
        return true;
    }
    else if (from == to) { /* base case 2: range of one element */
        return a[from] > 0;
    }
    else { /* recursive case */
        return a[from] > 0 && allPositiveHelper(a, from + 1, to);
    }
}
```



$$\text{isAllPos}(\frac{\{3, -1, 2\}}{a})$$

$$\downarrow$$
$$\text{allPos}(a, 0, 2)$$

$$\underline{a[0] > 0}$$

T

$$\underline{\text{allPos}(a, 1, 2)}$$

$$\underline{a[1] > 0}$$

T

$$\underline{\cancel{\text{allPos}(a, 2, 2)}}$$

int[] @ = new int[0];

all Positive (@)

|  
all PH (a, 0, -1)

from > to

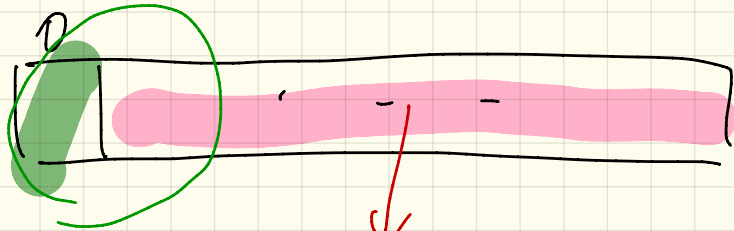


```
boolean allPositive(int[] a) { // if (a.length == 0) { return true }  
    return allPositiveHelper(a, 0, a.length - 1);  
}
```

```
boolean allPositiveHelper(int[] a, int from, int to) {  
    if (from > to) { /* base case 1: empty range */  
        return true;  
    }  
    else if (from == to) { /* base case 2: range of one element */  
        return a[from] > 0;  
    }  
    else { /* recursive case */  
        return a[from] > 0 && allPositiveHelper(a, from + 1, to);  
    }  
}
```

boolean  
isSorted (int[] a)

{ } → T



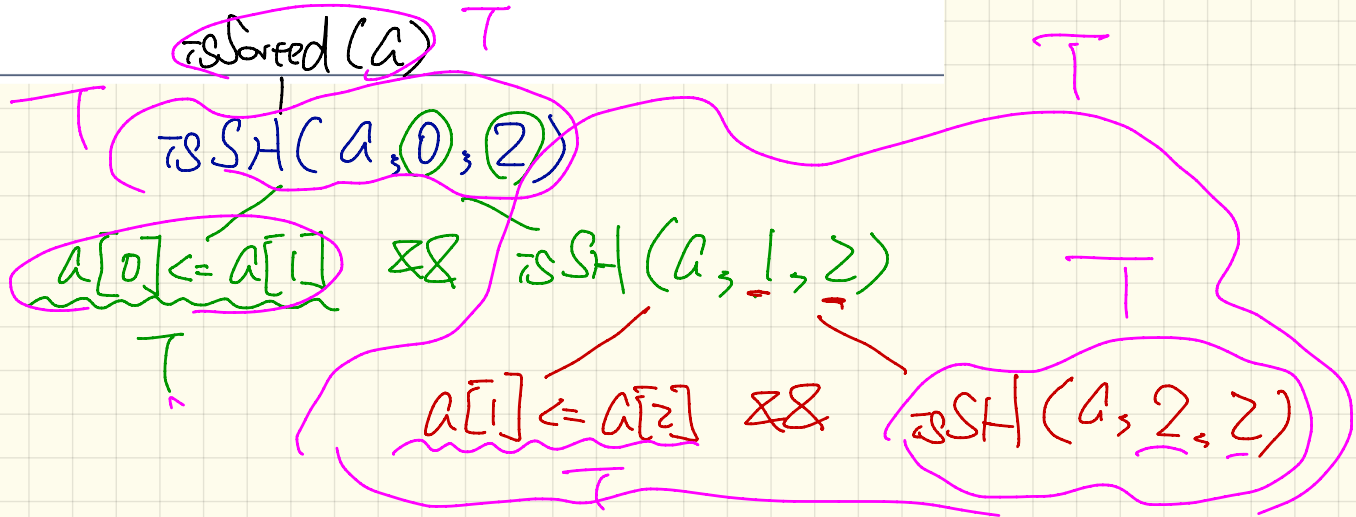
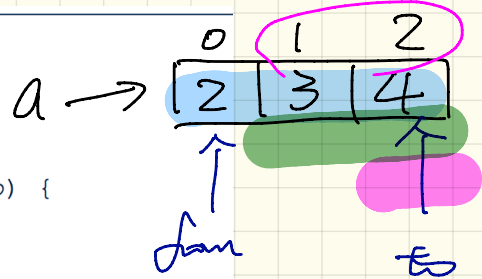
~~xx~~

isSorted ( )

$a[0] \leq a[1]$

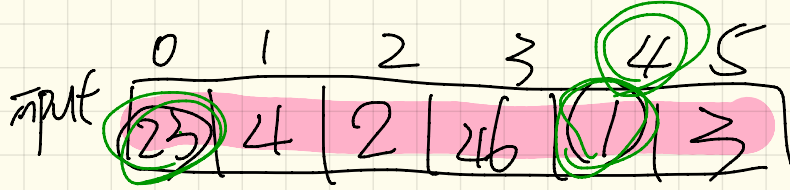
```
boolean isSorted(int[] a) {  
    return isSortedHelper(a, 0, a.length - 1);  
}
```

```
boolean isSortedHelper(int[] a, int from, int to) {  
    if (from > to) { /* base case 1: empty range */  
        return true;  
    }  
    else if (from == to) { /* base case 2: range of one element */  
        return true;  
    }  
    else {  
        return a[from] <= a[from + 1]  
            && isSortedHelper(a, from + 1, to);  
    }  
}
```

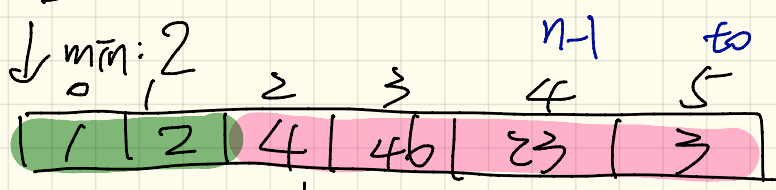
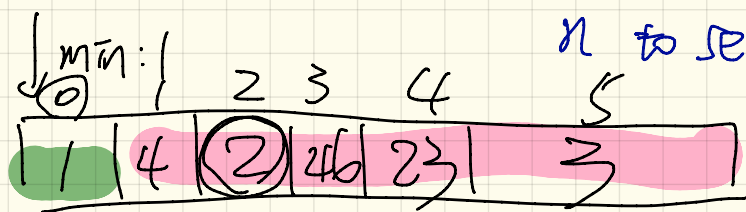


Lecture 18

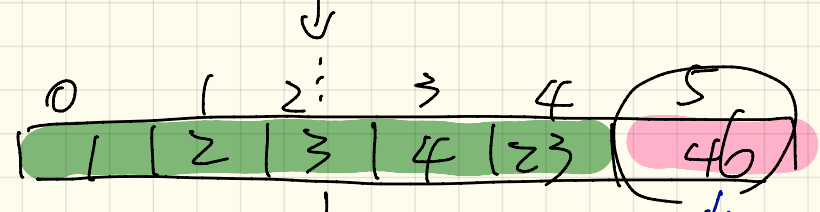
Thursday Nov. 9



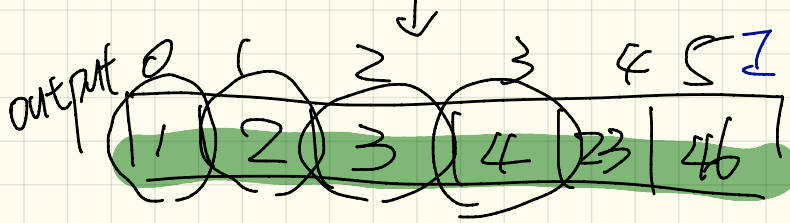
in-place sorting



$$n + (n-1) + \dots + 1 = O(n^2)$$



to select the last times



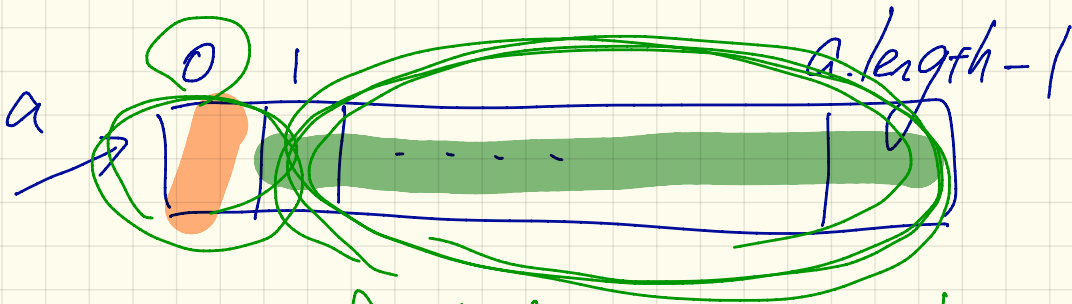
Problem: Recursively find min from an array  
(assume  $a.length \geq 1$ )

int findMin (int[] a)

Base Case

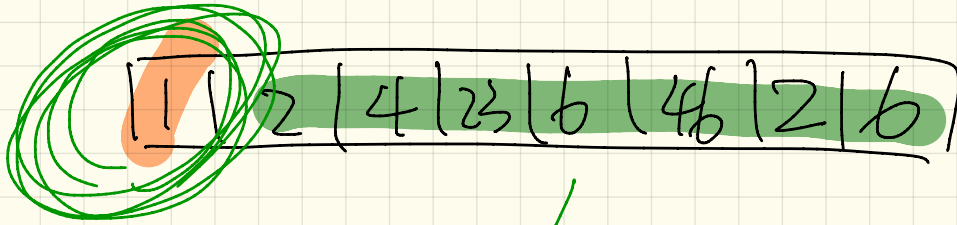
$a.length == 1 \rightarrow$

return  
 $a[0]$

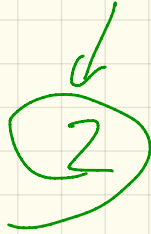


$\text{orange} < \text{findMin}(a, 1, a.length-1)$

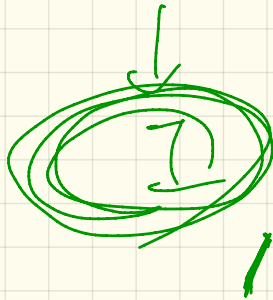
$\hookrightarrow$  min is  $\text{orange}$



index (0)

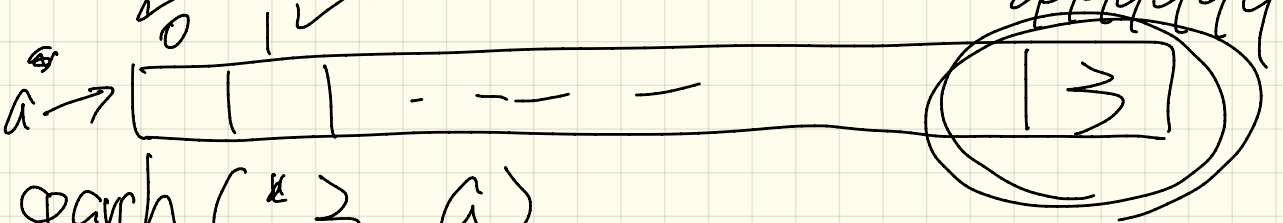


4



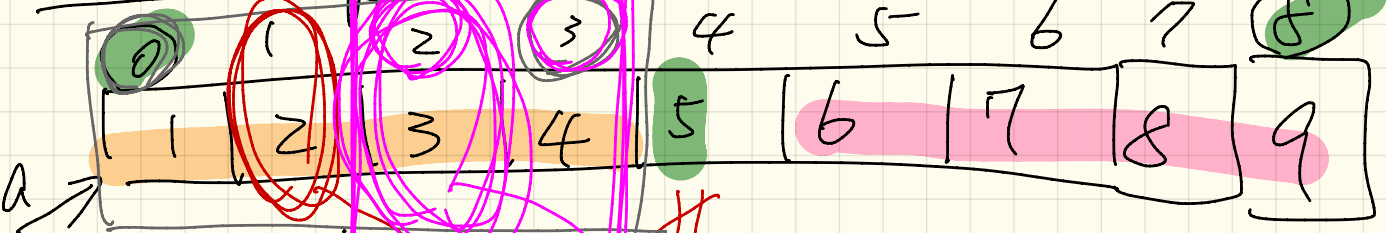
index (4)

unsorted array



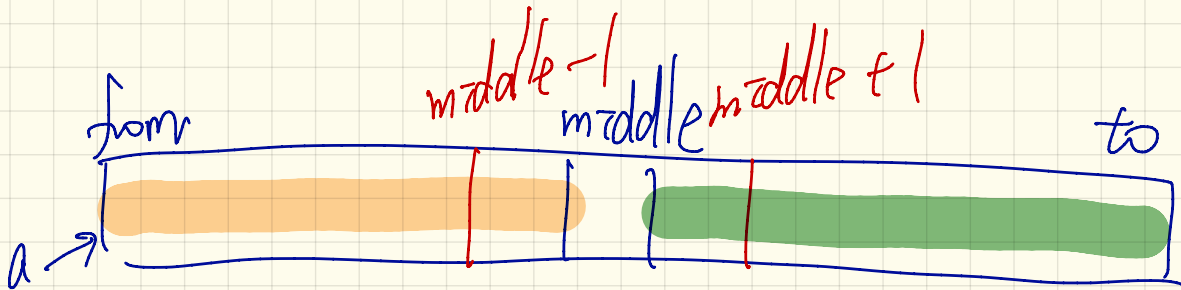
$O(n)$

sorted array



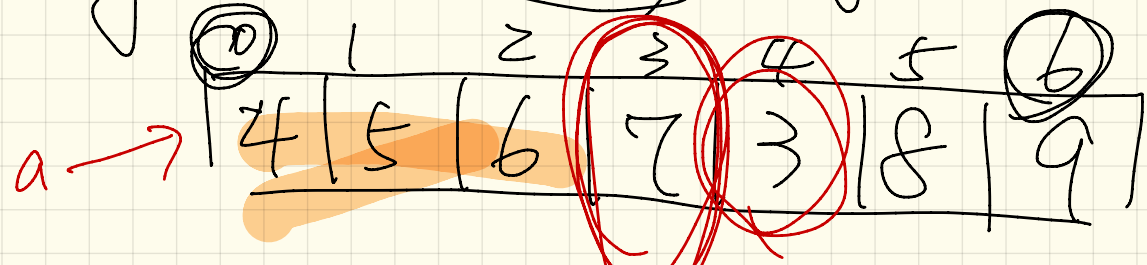
search (3, a)  $\neq 3$   $\rightarrow$   $\neq 3$   $\rightarrow$  true.



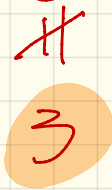


key  
key < a[middle]

binary search on unsorted array



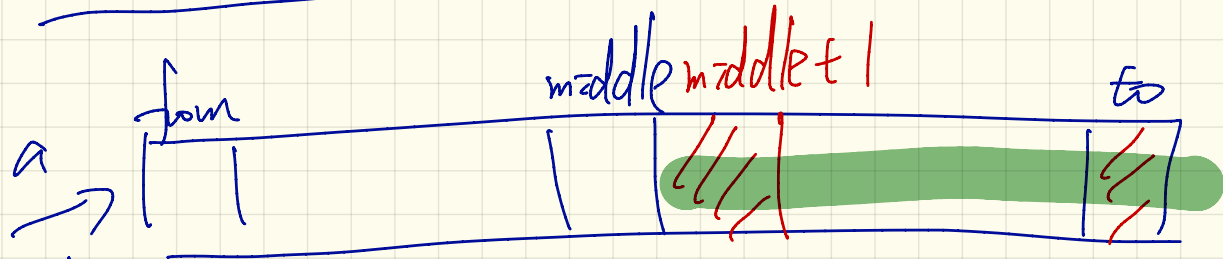
search(3, a)

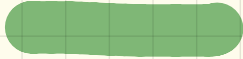


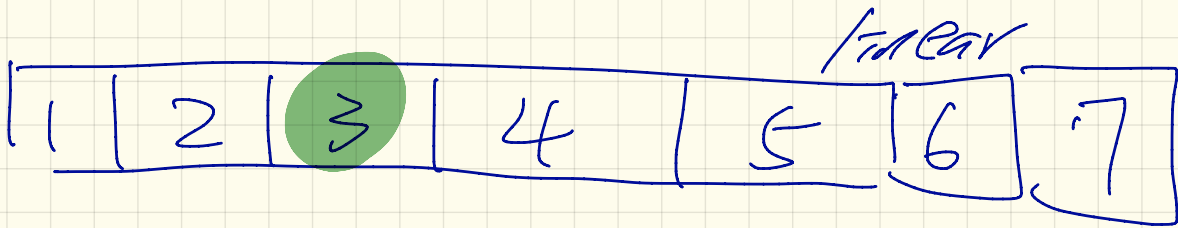
↓  
false (wrong!!)

How <sup>to</sup> do binary search if  
the array is sorted in descending  
order?

---

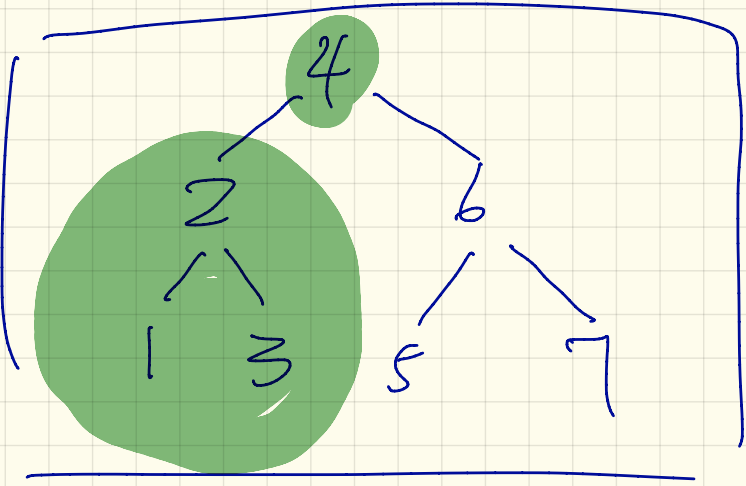


$key < a[middle] \rightarrow$  



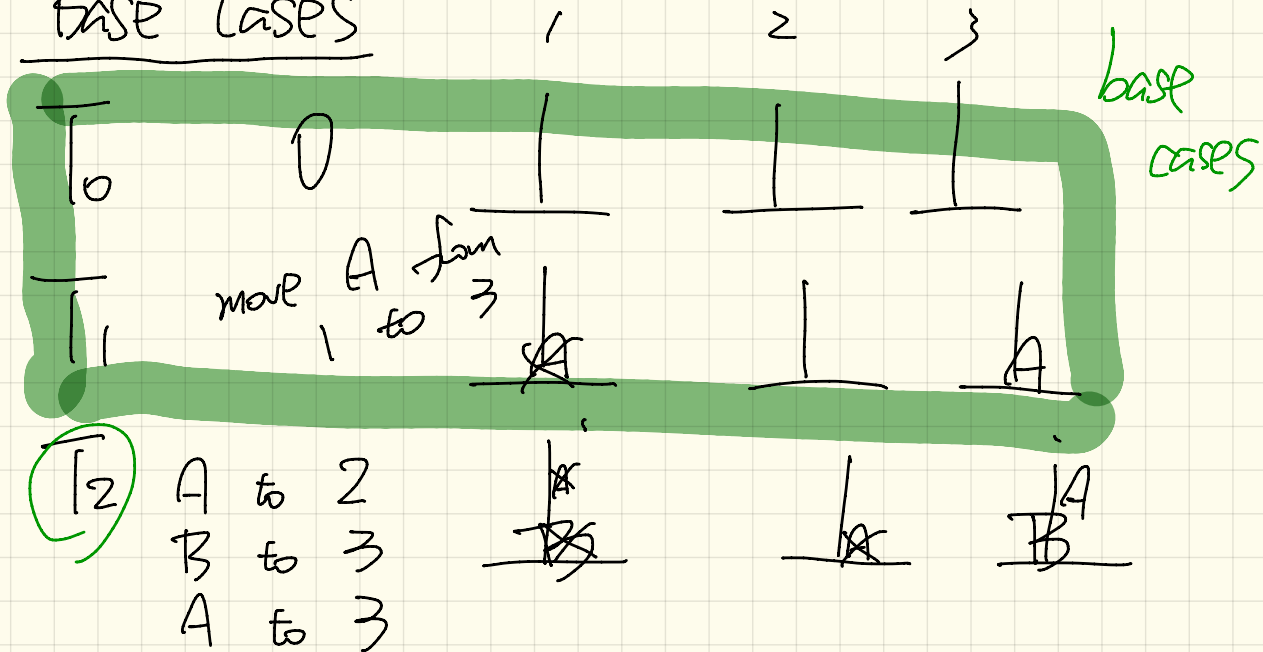
Binary-search-tree

non-linear

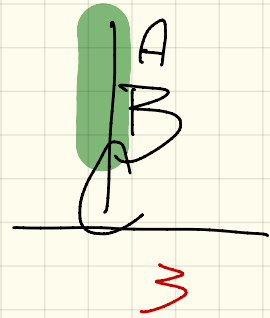
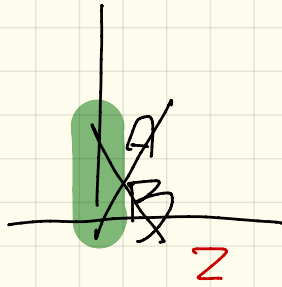
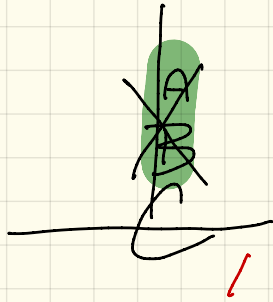
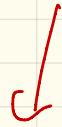


# Tower of Hanoi (move from peg 1 to peg 3)

## Base Cases



I 3



move  
A

B  
C from 1 to 3

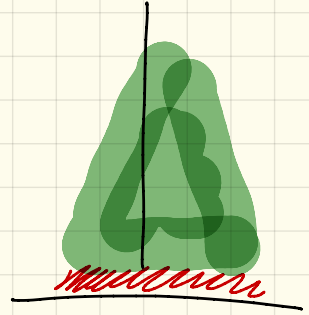
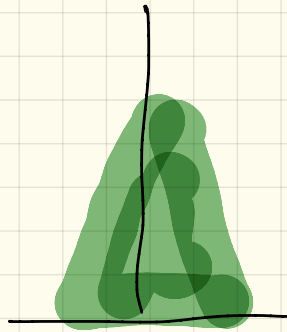
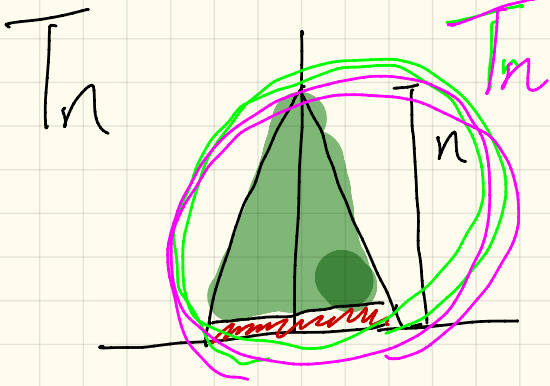
move A  
B from 1 to 2?  
3?

3 Steps:

Move A  
B from I to 2

Move C from I to 3

Move A  
B from 2 to 3



3 Steps

✓ - Move



- Move



- Move



$$T_0 = 0$$

$$T_1 = 1$$

$$T(n) = T(n-1) + 1 + T(n-1)$$

from 1 to 2  $2 \cdot T(n-1) + 1$

from 1 to 3 1

from 2 to 3  $T(n-1)$

$$\begin{cases} - \overline{T}(0) = 0 \\ - \overline{T}(n) = 2 \cdot \overline{T}(n-1) + 1 \end{cases}$$

$$T(4) = 2 \cdot T(3) + 1$$

$$= 2 \cdot (2 \cdot T(2) + 1) + 1$$

$$= 2 \cdot (2 \cdot (2 \cdot T(1) + 1) + 1) + 1$$

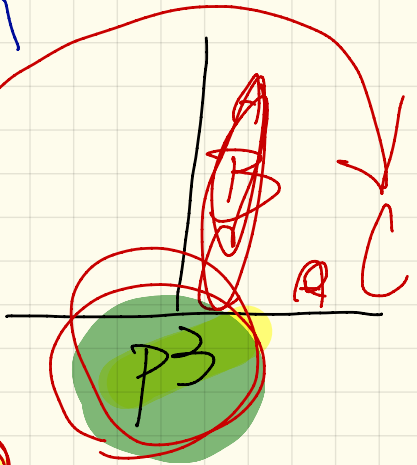
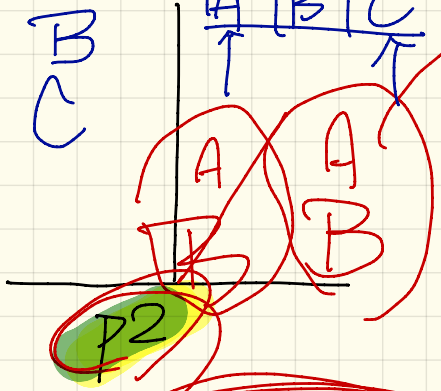
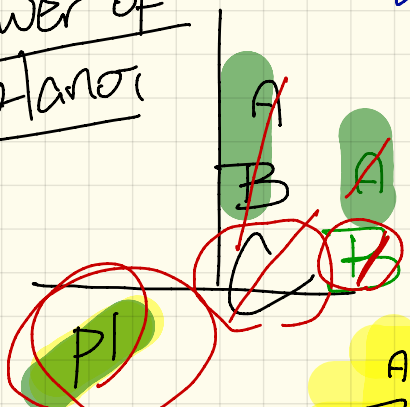


Lecture 19

Tuesday Nov. 14

# Tower of Hanoi

tower: A  
B  
C



Problem: Move C from p1 to p3

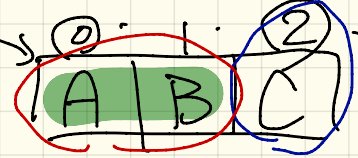
✓ Move A from p1 to p2

✓ Move C from p1 to p3

✓ Move B from p2 to p3

Move A from p1 to p3  
Move B from p1 to p2  
Move A down p3 to p2

Problem: Move  $\begin{matrix} A \\ B \\ C \end{matrix}$  from  $p1$  to  $p3$  <sup>ds</sup>



tohH(ds, 0, 2, p1, p3) =  
 {A, B, C}

intermediate = 6 - 1 - 3

= 2

intermediate: p3

tohH(ds, 0, 0, p1, p3)

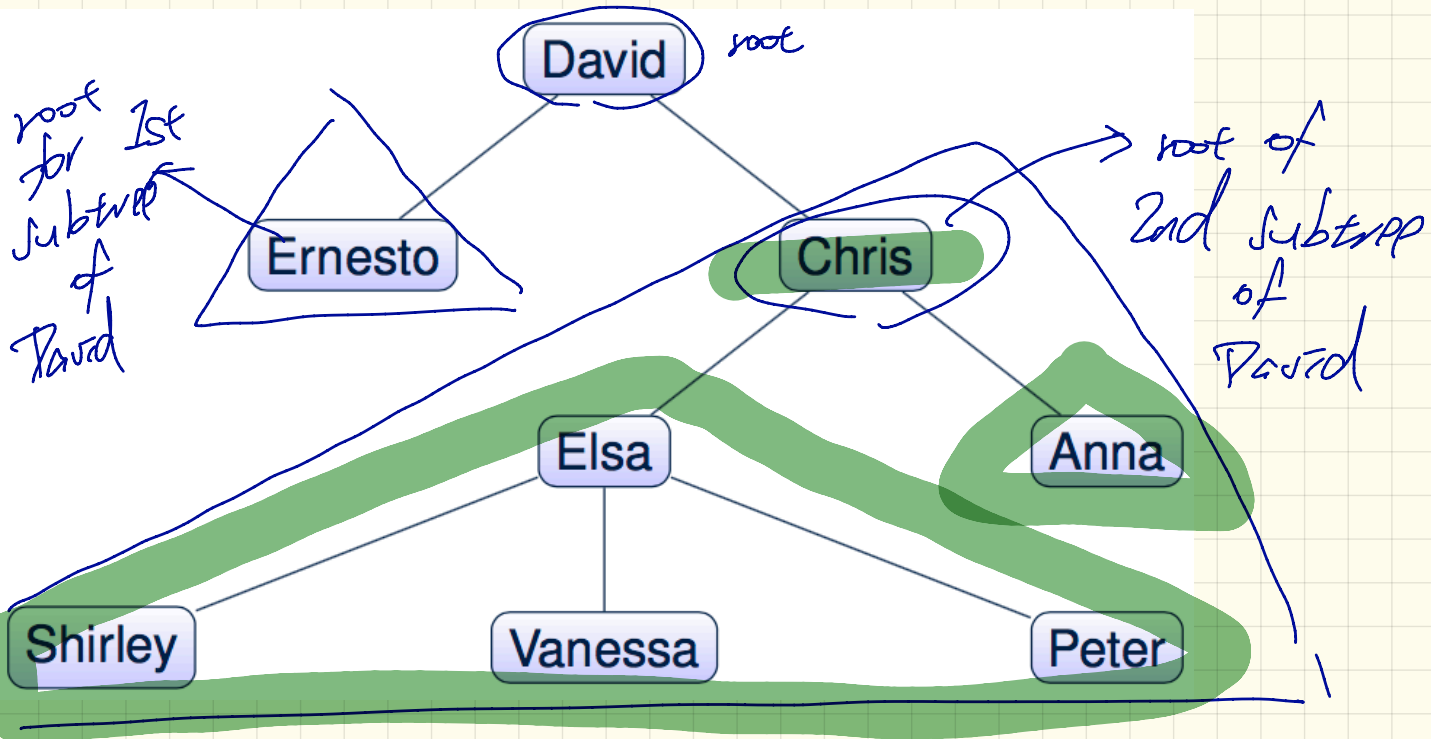
move ds[0] from p1 to p2

tohH(ds, 0, 0, p3, p2)

tohH(ds, 0, 1, p1, intermediate)  
 {A, B}

Move ds[to] from p1 to p3

tohH(ds, 0, 1, intermediate, p3)  
 {A, B}

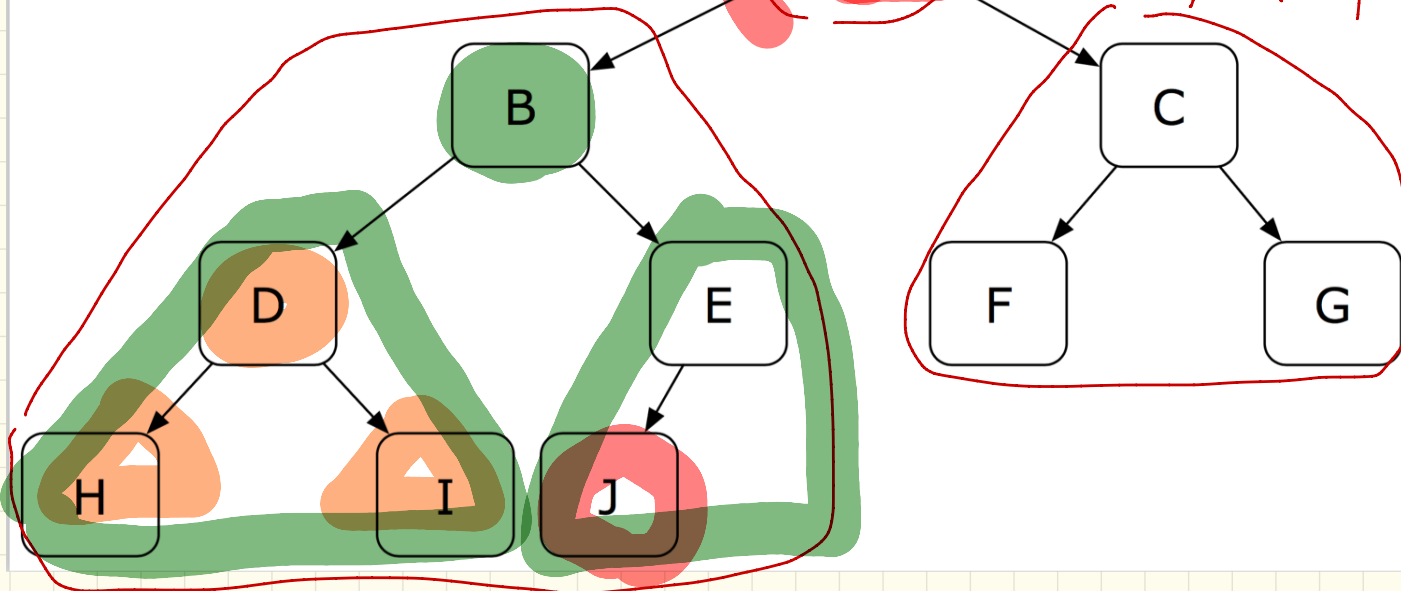


LST of A

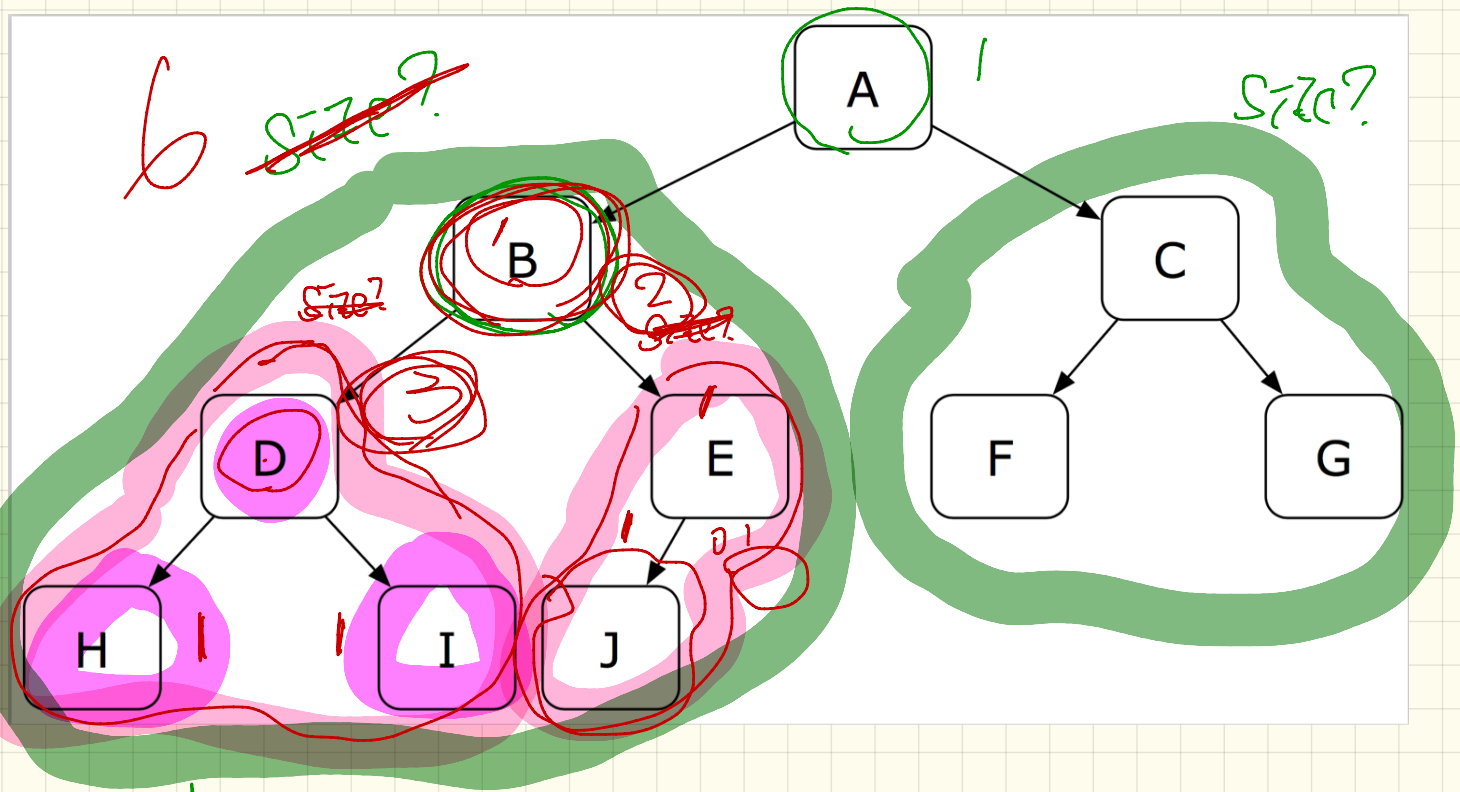
A

root

RST of A



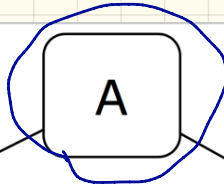
6 size?



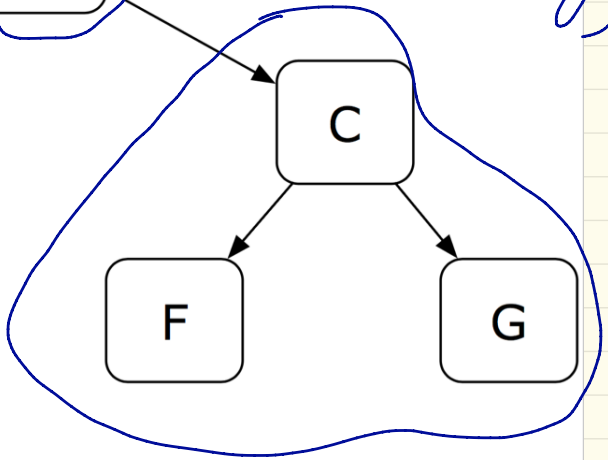
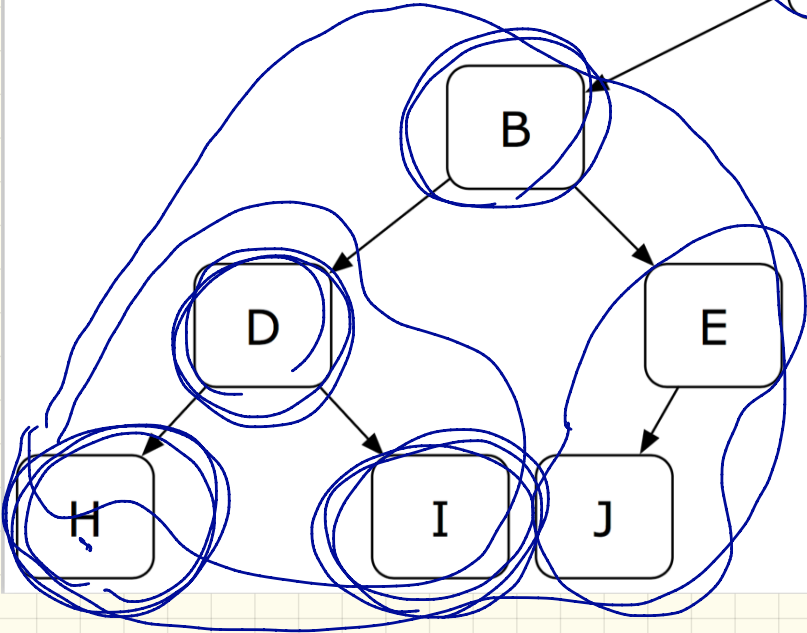
size?

Size of subtree rooted at A

? has(A.left)



? has(A.right)

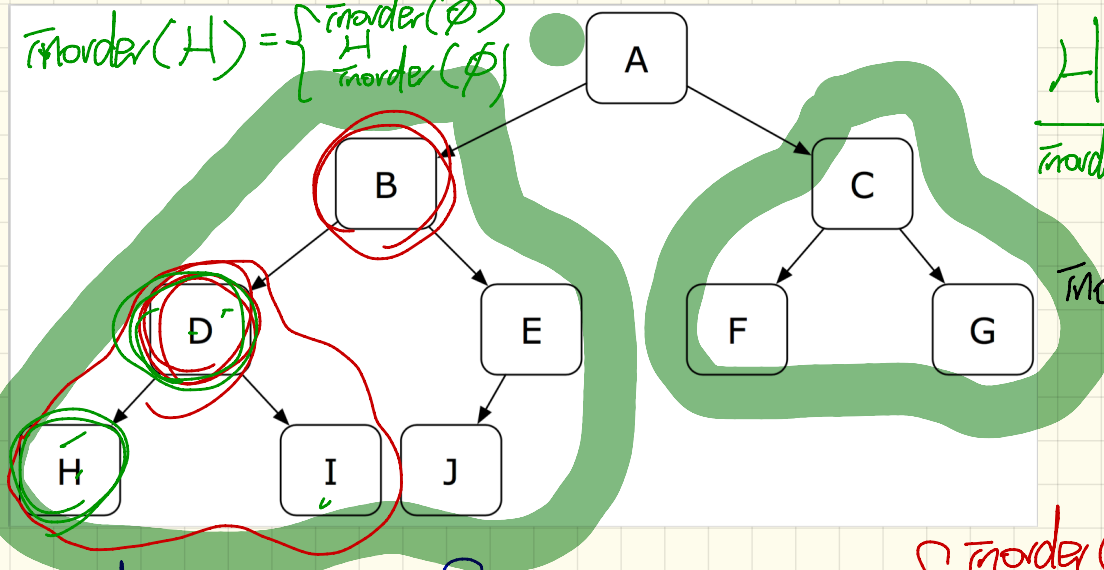


has(H)

has(A) =

A == H ||  
has(A.left) ||  
has(A.right).

$$\text{inorder}(H) = \begin{cases} \text{inorder}(\emptyset) \\ H \\ \text{inorder}(\emptyset) \end{cases}$$



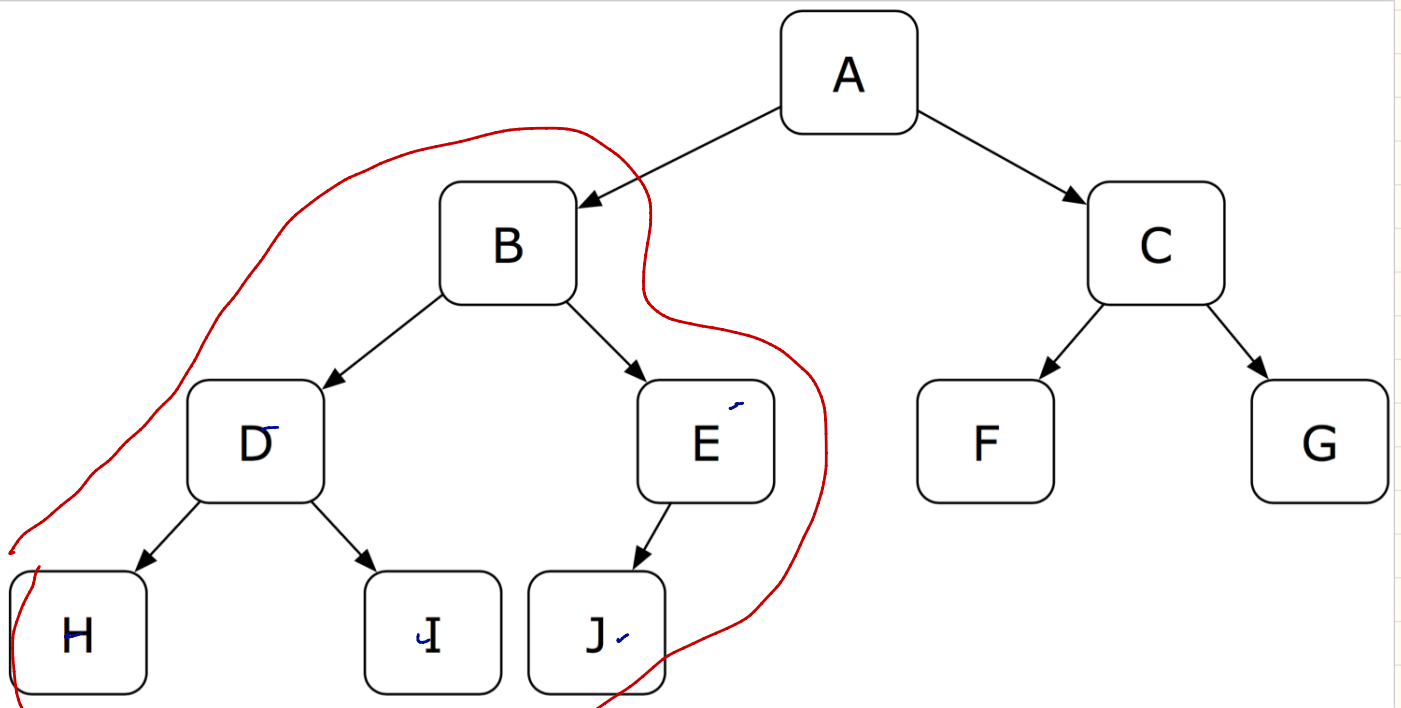
HDI BJE  
 $\text{inorder}(D)$        $\text{inorder}(E)$

$\text{inorder}(A)$

$$\text{inorder}(A) = \left\{ \begin{array}{l} \text{inorder}(A.\text{left}) \\ \underline{B} \\ \text{inorder}(A.\text{right}) \end{array} \right\} A$$

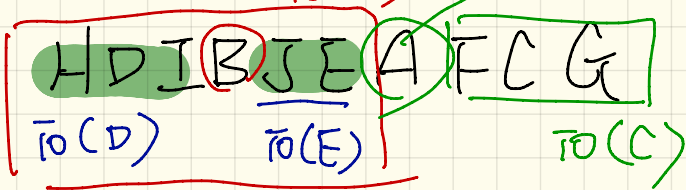
$$\left\{ \begin{array}{l} \text{inorder}(B.\text{left}) \\ \underline{B} \\ \text{inorder}(B.\text{right}) \end{array} \right\} \left\{ \begin{array}{l} H \\ \text{inorder}(H) \\ \underline{D} \\ \text{inorder}(I) \end{array} \right\}$$





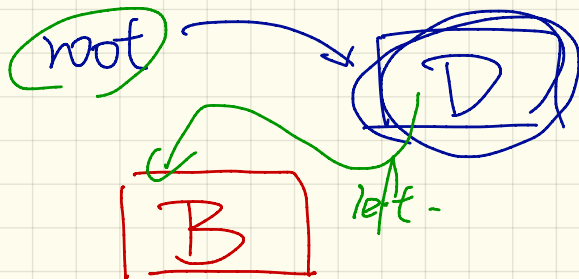
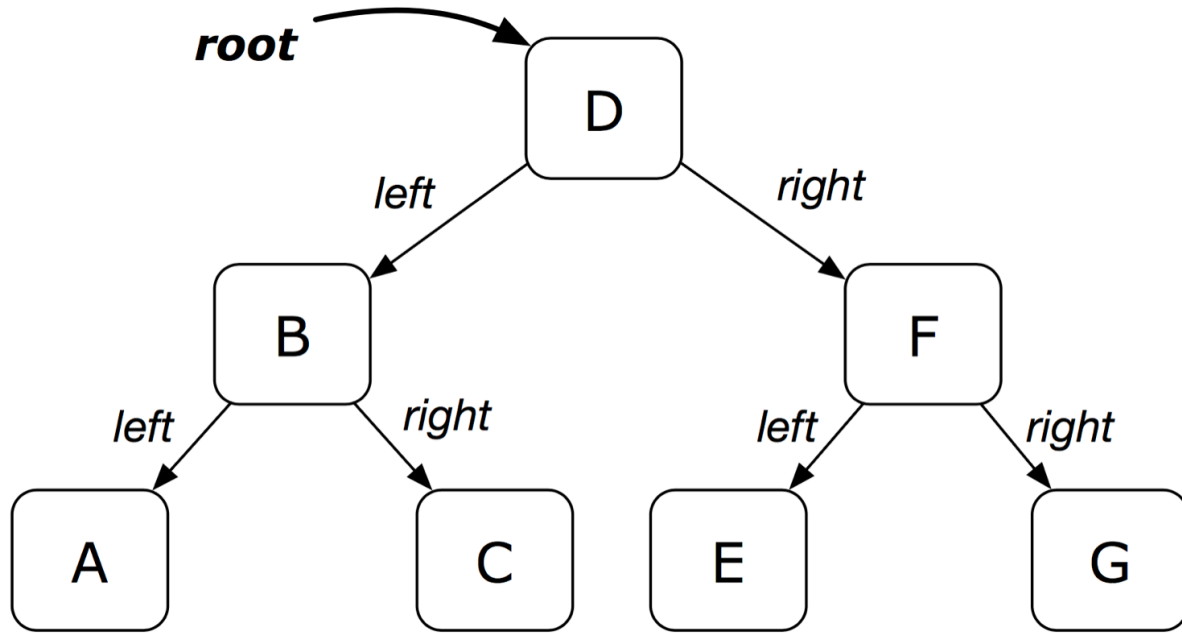
$\text{tr inorder}(A)$

$\text{tr}(A)$



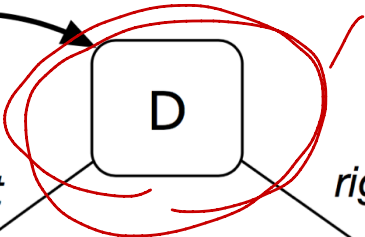
Lecture 20

Thursday Nov. 16



`bt.addToLeft(root, "B")`  
↳ `root.setLeft(new BTNode("B"))`

root



left

right

$$\text{size}(D) =$$

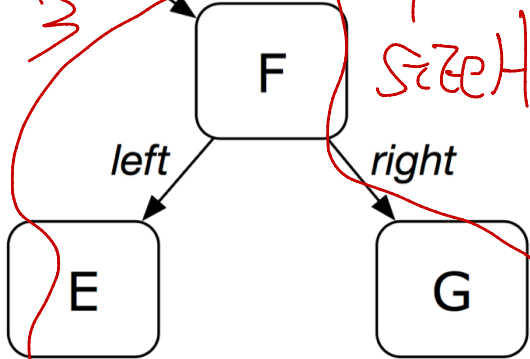
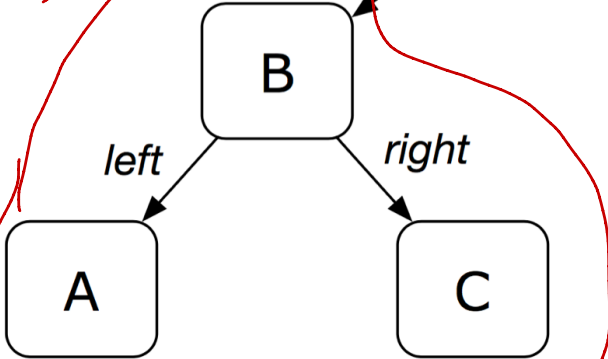
$$\text{size}(D.\text{left})$$

$$+ \text{size}(D.\text{right})$$

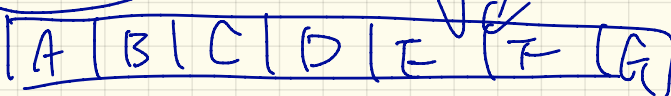
$$+ 1$$

3

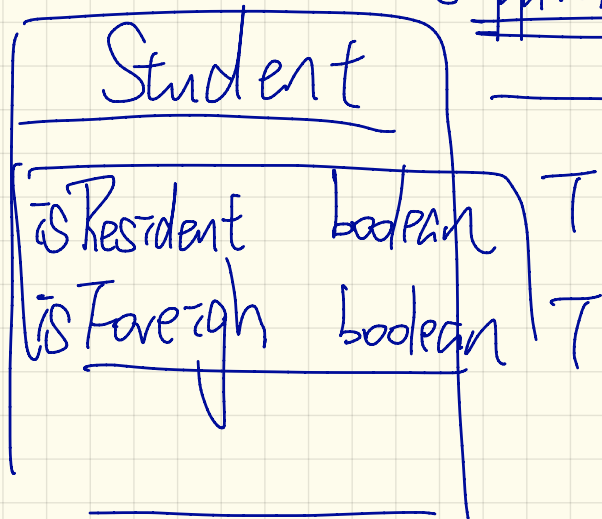
3



(bt. inorder()) → ArrayList



Supplier



Use

Student s = \_\_\_\_\_

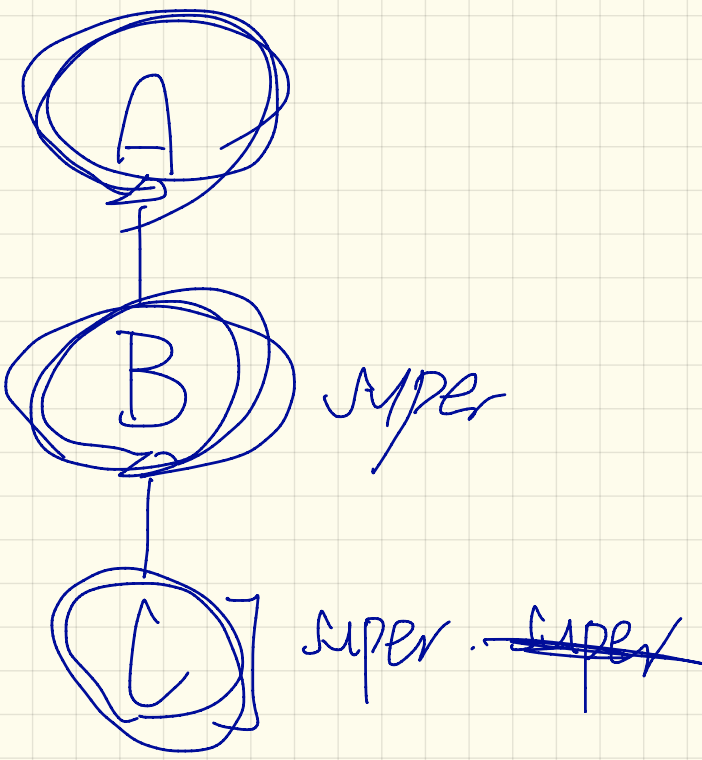
m(s);

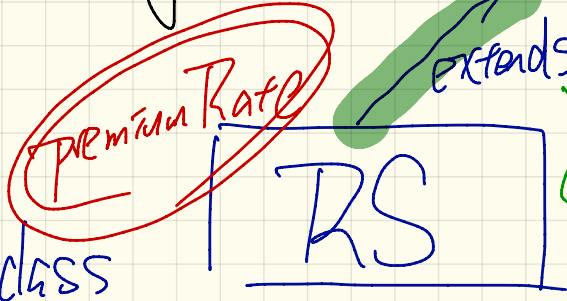
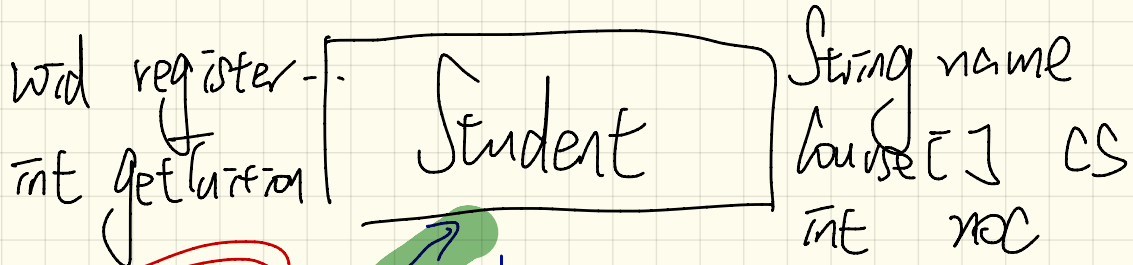
```
m() {  
    if (s.isForeign) {  
        if (s.isResident) {  
            } else {  
            }  
        }  
}
```

# Single Choice Principle

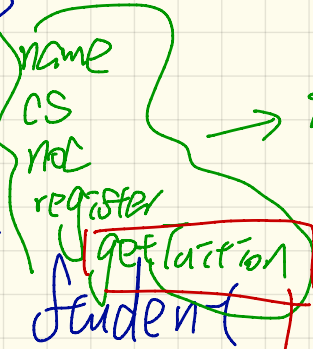
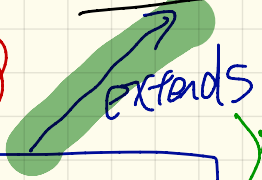
---

↳ when there's a change,  
there should be only one  
place (class, method) to apply the change.





Resident Student extends



inherited, no need to ~~repeat~~ them.

override/redefine



# Resident Student vs ;

Student

STATIC type

defines the list of EXPECTATIONS on 'S'

S =  
expectations of S

- S. NAME
- S. NOC
- S. VCS
- S. REGISTER (..)
- S. GETLATION (..)

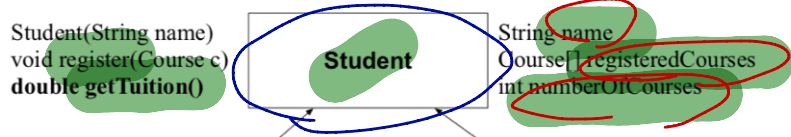
noe expectations of S. [ S. Pr  
S. dv

inherits All expectations from Student

addressed is pr → expect.

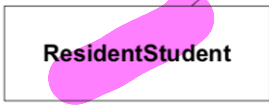
Lecture 21

Tuesday Nov. 21



```

/* new attributes, new methods */
ResidentStudent(String name)
double premiumRate
void setPremiumRate(double r)
/* redefined/overridden methods */
double getTuition()
  
```



```

/* new attributes, new methods */
NonResidentStudent(String name)
double discountRate
void setDiscountRate(double r)
/* redefined/overridden methods */
double getTuition()
  
```

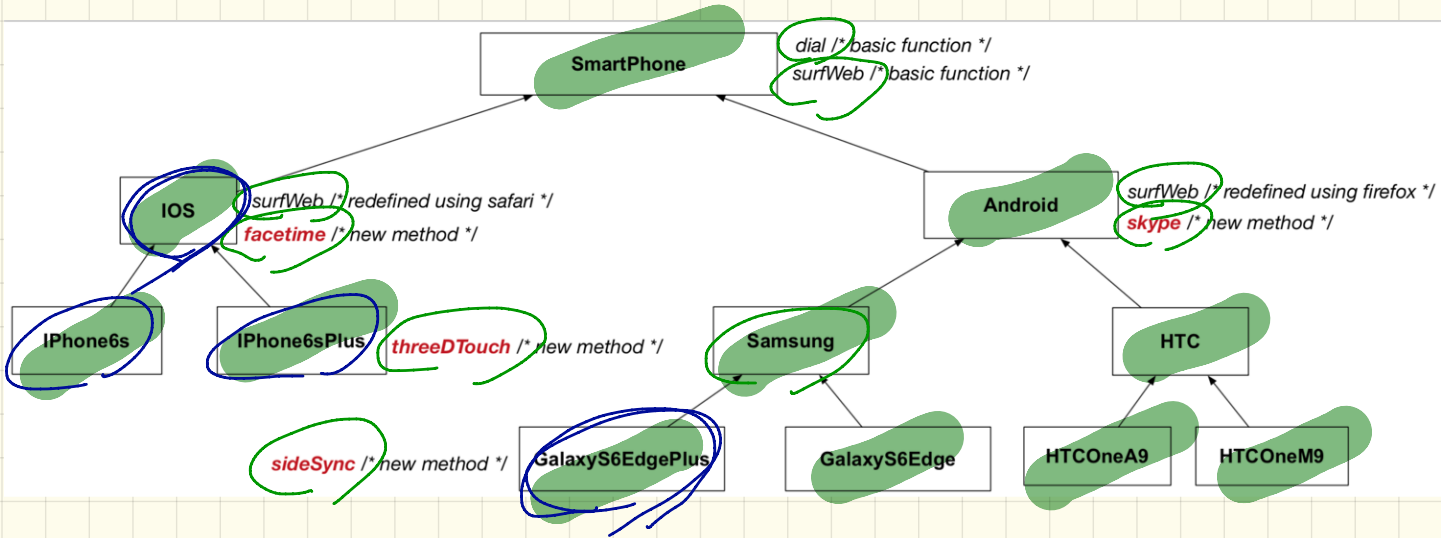
→ static type

```

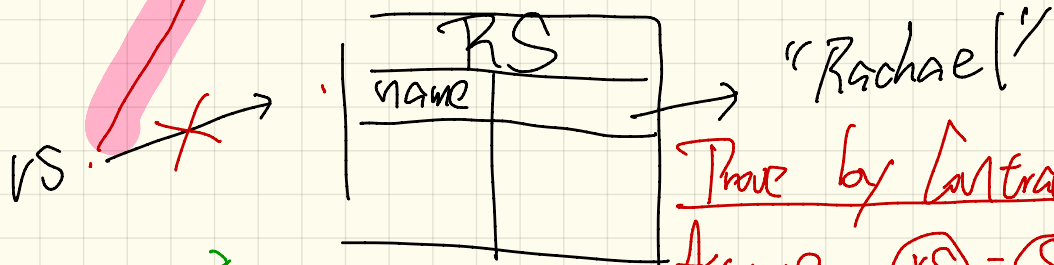
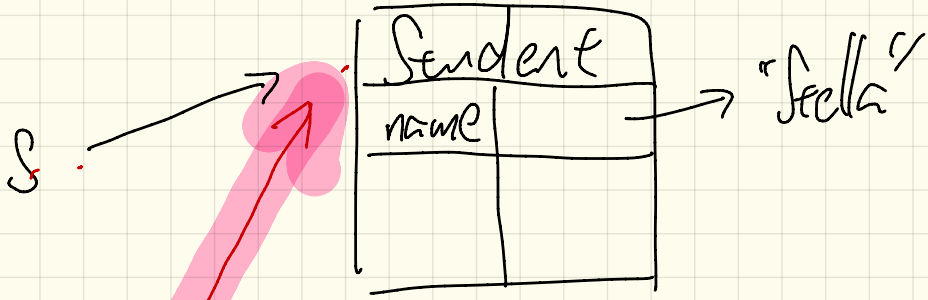
Student s = new Student("Stella");
ResidentStudent rs = new ResidentStudent("Rachael");
NonResidentStudent nrs = new NonResidentStudent("Nancy");
  
```

	name	rCs	noC	reg	getT	pr	setPR	dr	setDR
s.			✓					×	
rs.		✓				✓			×
nrs.			✓				×		✓

expectations



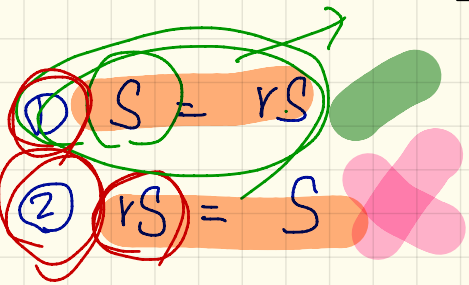
IOS myPhone ;



Proof by Contradiction

$S = rs$  ① Assume  $(rs = S)$  compiled

② Expectation for  $rs$ ? → Resid. Stu.  
 $rs.premiumRate$   
 → crash ∵ undefined on  $Stu.$  object.



RS

RS	
name	"Rachael"
dv	125

dynamic type  
"Rachael"

125

S

~~S = nrs~~

Sender

nrs

nrs	
name	"Nancy"
dv	0.75

dynamic type

75

~~Student~~  $s \rightarrow$   $\boxed{S}$   
 $(S) = \text{new Student}(\dots);$   
 $s = \text{get tuition}();$   
 ResidentStudent vs = new RSC(...);

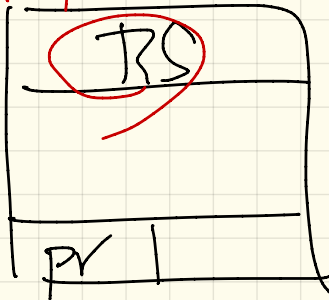
$(S) = \text{rs};$

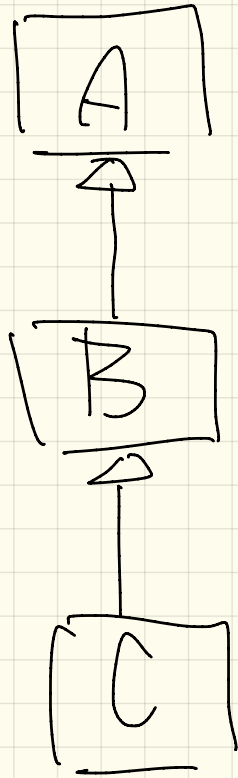
~~S. premiumRate~~ ?

does not compile

Student S

ST of S (Student) does not declare premiumRate.





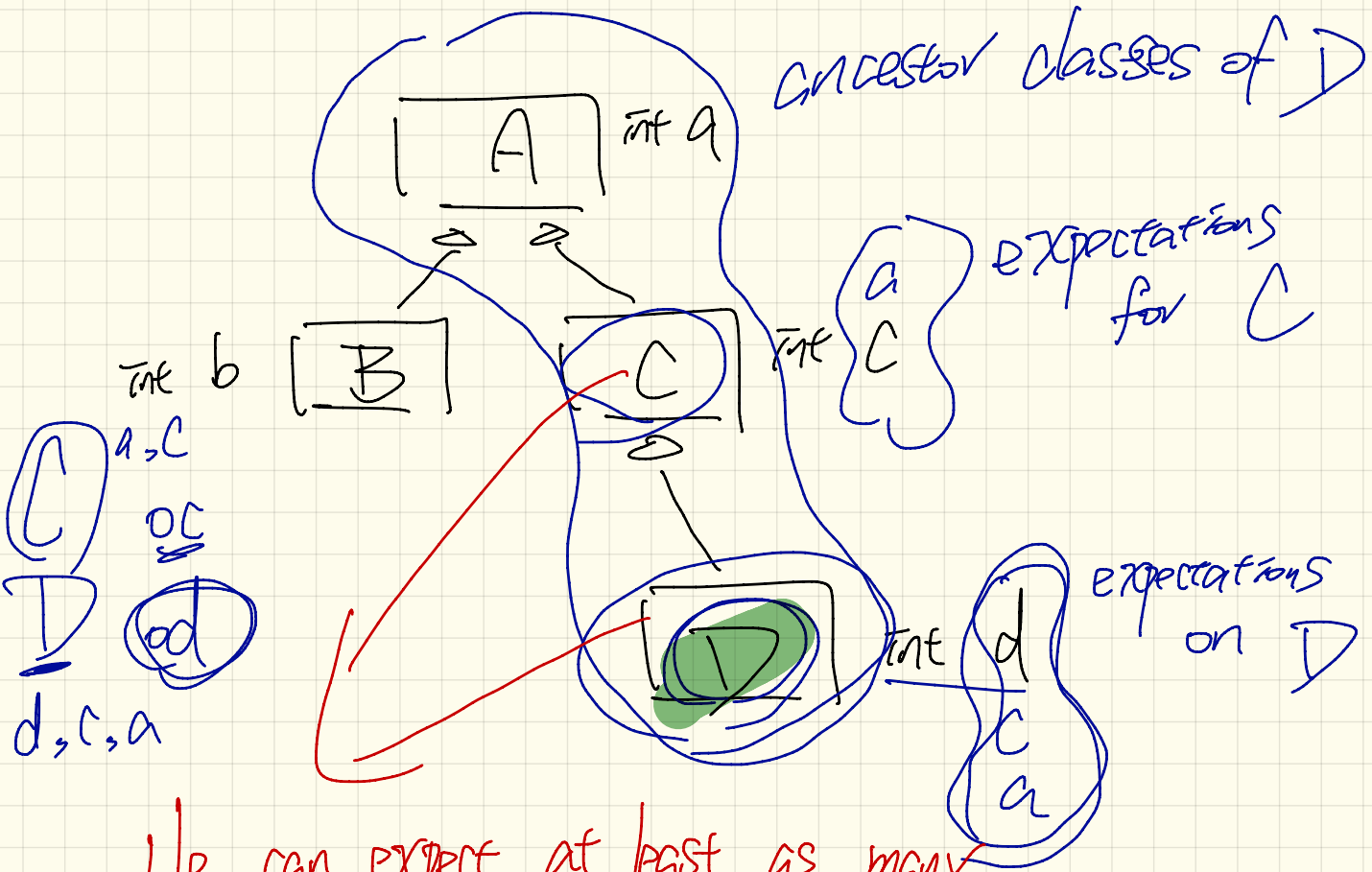
$C$  implicitly extends  $A$

---

$a > b \wedge b > c$

$a > c$





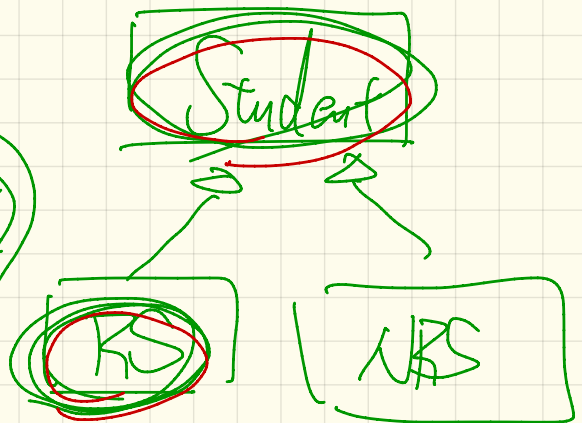
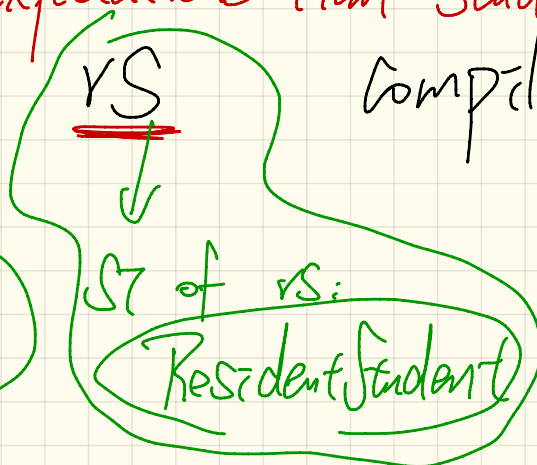
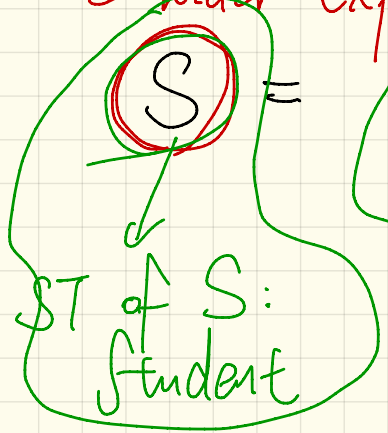
We can expect at least as many on D, compared with C.

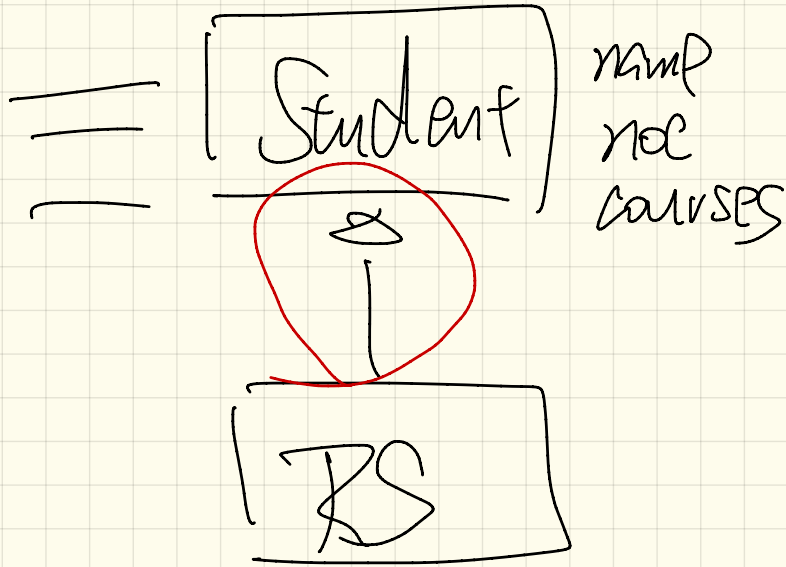
Student  $S = \underline{\underline{rs = S}}$

ResidentStudent  $rs = \underline{\underline{\quad}}$

wider expectations than Student (pr)

compiles ✓





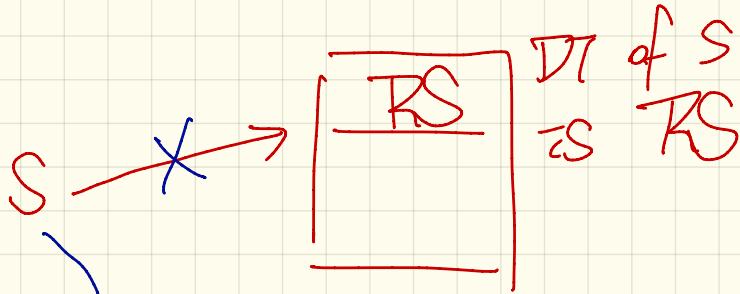
Student  $S = \text{---}$

Res: Stu  $rs = \text{---}$

$rs = S ;$  ~~X~~

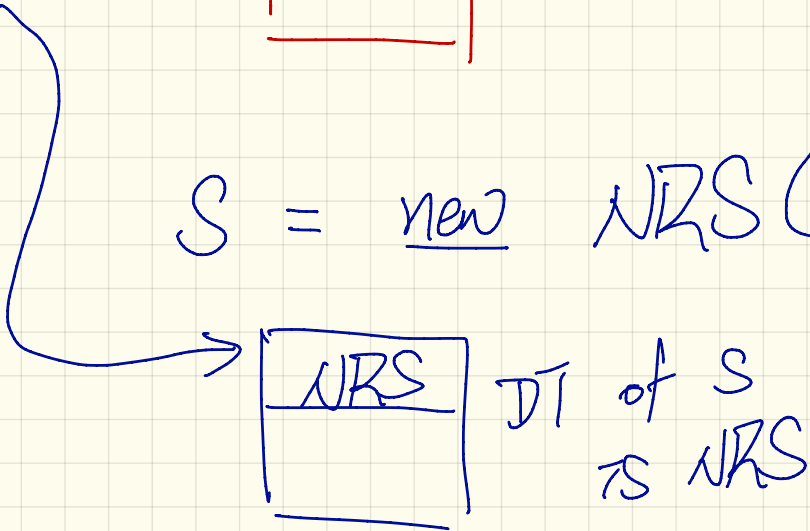
Student

S = new RS(--);



new DT of S

S = new NRS(---);



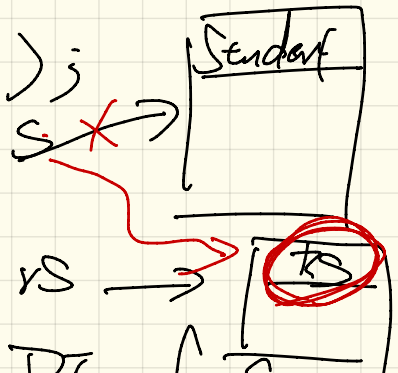
① Student s = new Student();

② Student rs = new RS();

③ Student nrs = new NRS();

④ Student s = rs; → ST: Student

⑤ s = nrs;



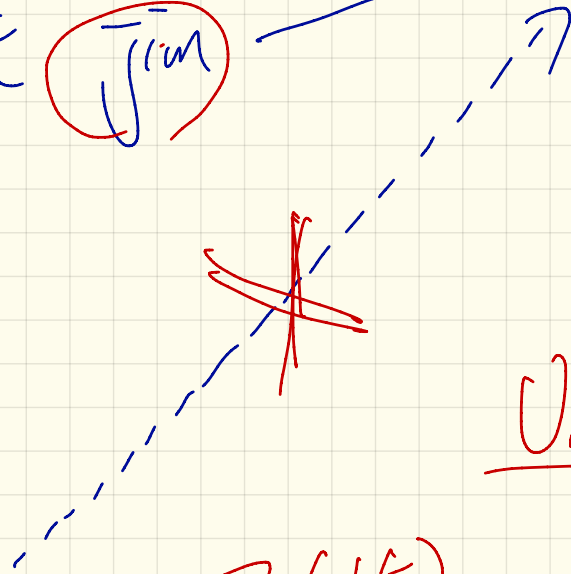
	ST of s	DT of s
①	Student	Student s.get(C)
②	Student	Student
③	Student	Student
④	Student	RS s.get(C)
⑤	Student	NRS s.get(C)

Student

Jim



TRS	
pr	1.5

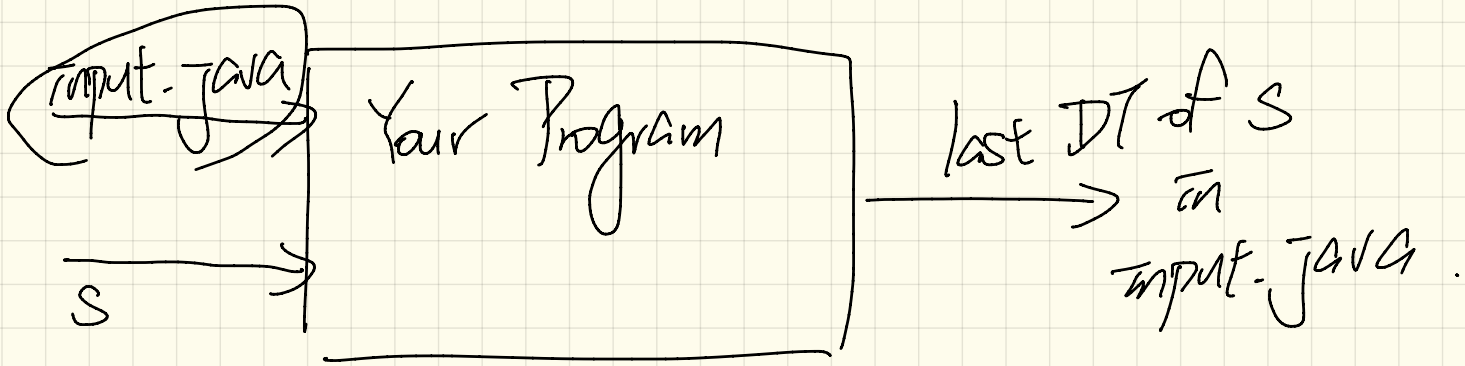


RS

rs

rs set Pr(1.5)

Undecidability



```
input.java  
-----  
Student s = null;  
while( true ) {  
    }  
s = new RC(...);
```

Lecture 22

Thursday Nov. 23



Student(String name)  
void register(Course c)  
**double getTuition()**



String name  
Course[] registeredCourses  
int numberOfCourses

*/\* new attributes, new methods \*/*  
**ResidentStudent**(String name)  
double premiumRate  
void setPremiumRate(double r)  
*/\* redefined/overridden methods \*/*  
double getTuition()

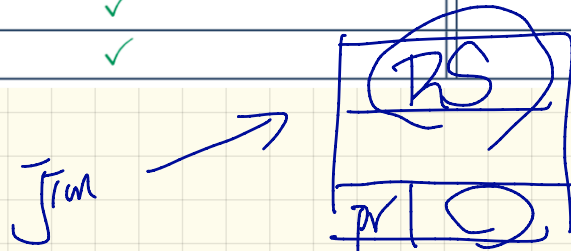


*/\* new attributes, new methods \*/*  
**NonResidentStudent**(String name)  
double discountRate  
void setDiscountRate(double r)  
*/\* redefined/overridden methods \*/*  
double getTuition()

```

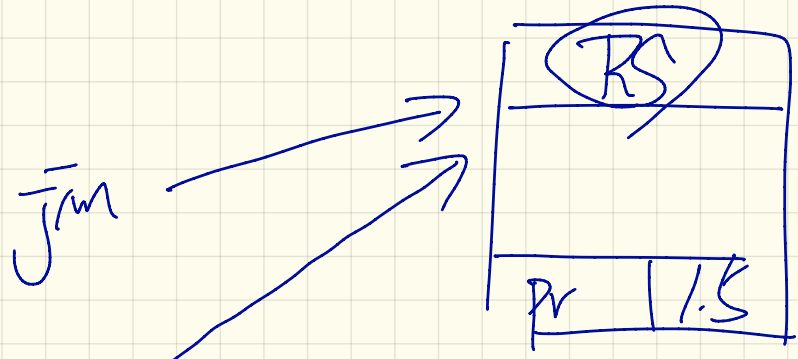
Student s = new Student("Stella");
ResidentStudent rs = new ResidentStudent("Rachael");
NonResidentStudent nrs = new NonResidentStudent("Nancy");
  
```

	name	rCS	noC	reg	getT	pr	setPR	dr	setDR
s.			✓					✗	
rs.			✓				✓		✗
nrs.			✓				✗		✓



\* ResidentStudent IS = (ResidentStudent) [Jim]

ST: ResidentStudent



ST: Student

(RS) Jim. setPremiumRate(1.5) X

ST: Student

Student  $\bar{j}r_m = \overset{\text{new}}{RS}(\dots);$

① Resident student  $\underline{rs} = (\underline{RS}) \bar{j}r_m;$   
 $\underline{rs}. \text{setPr}(1.5);$

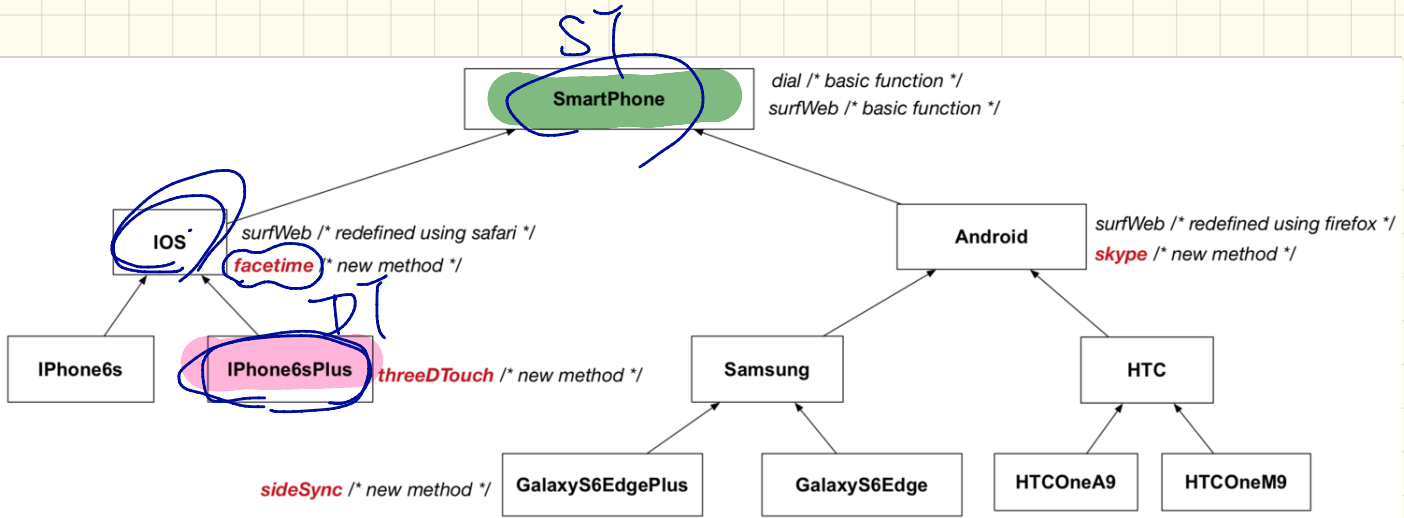
②  $(\underline{RS}) \bar{j}r_m). \text{setPr}(1.5)$

③  $(RS) \{ \bar{j}r_m. \text{setPr}(1.5) \};$

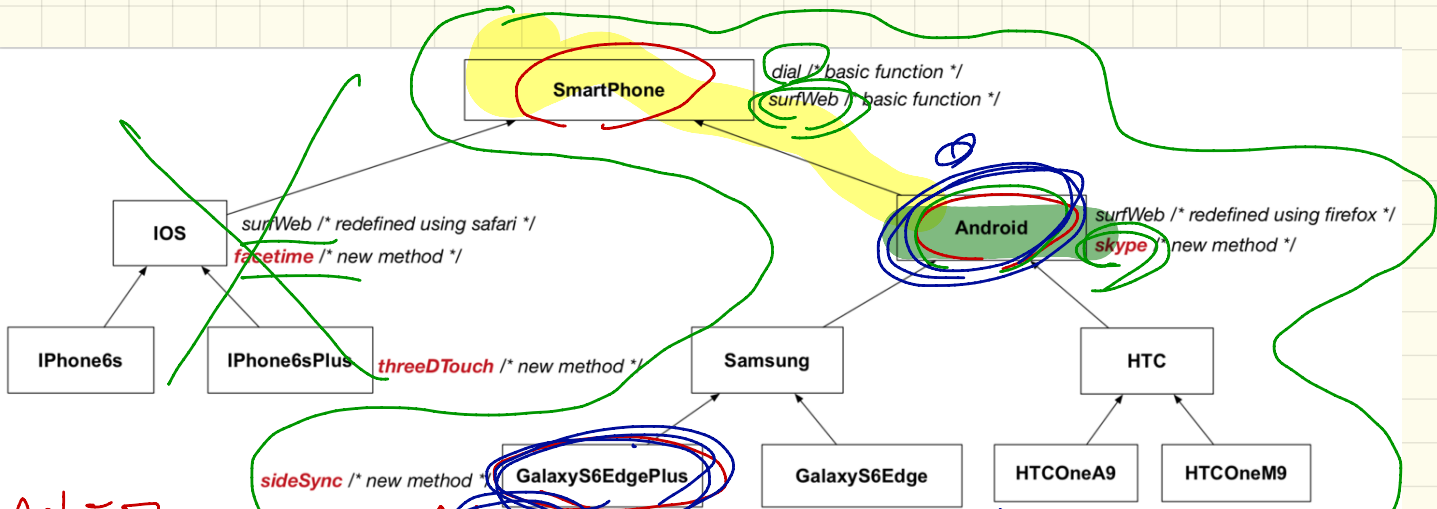
Student  $\bar{j}m = \underline{\text{new}} \text{RSC}(\dots);$

Student  $S;$

$\frac{S}{\downarrow}$  =  $\frac{\bar{j}m}{\downarrow}$   
Student      ST: Student

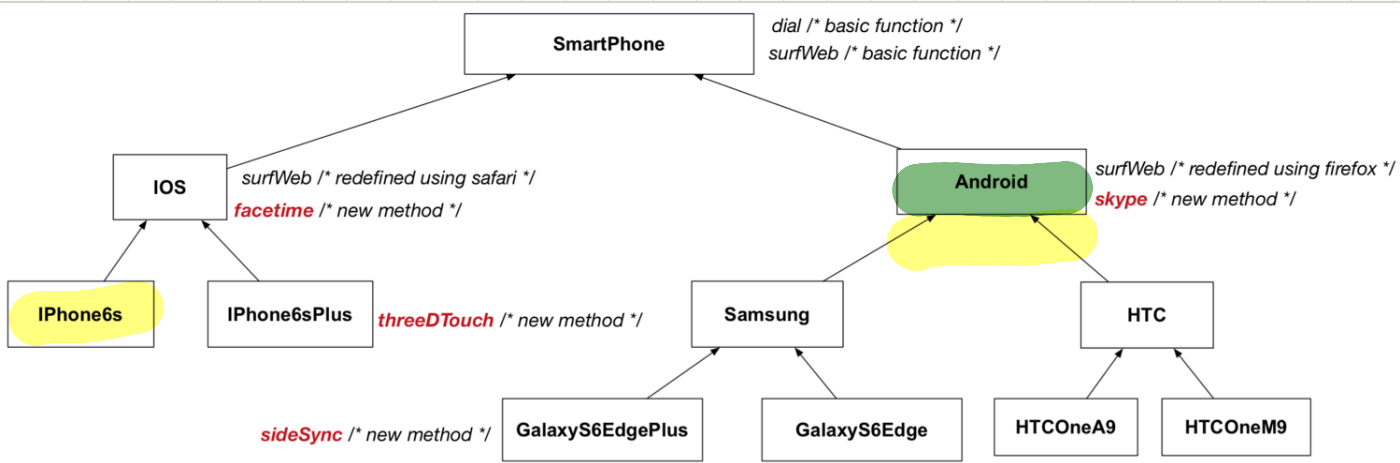


? IOS further year = aPhone;  
 further year → IP6sPlus  
 ↓  
 SP?



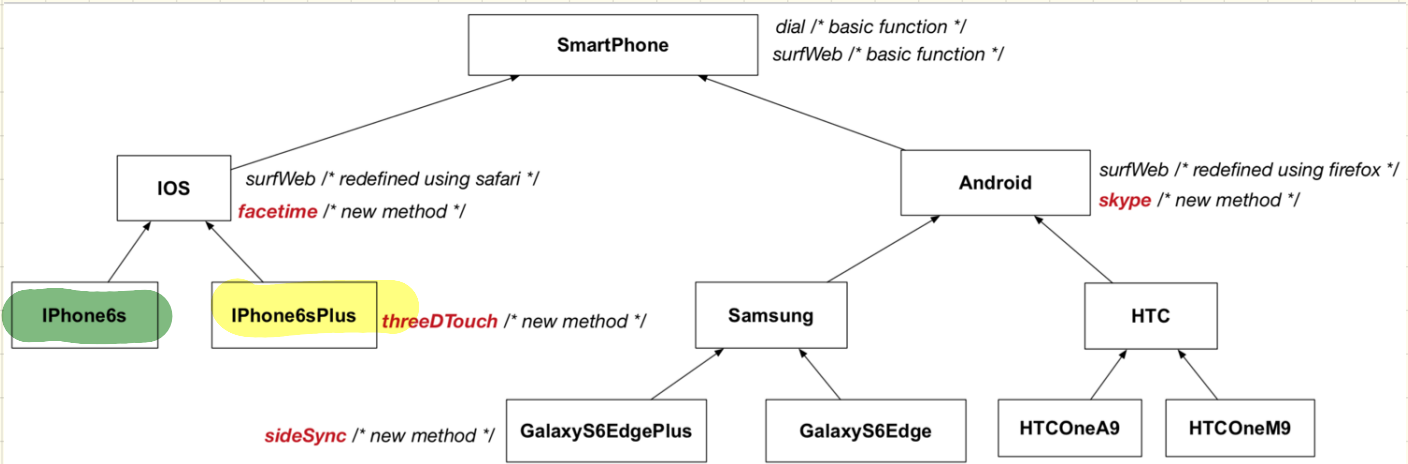
~~ga = (SmartPhone) phone; → ST: Android  
 Android phone = [ --- ];  
 phone.dial surfWeb skype~~

SmartPhone sp = (SmartPhone) phone; → ST: Android  
 sp.dial surfWeb skype



Android phone = X

iPhone6s ipbs = (iPhone6s) phone ;  
 ST: Android

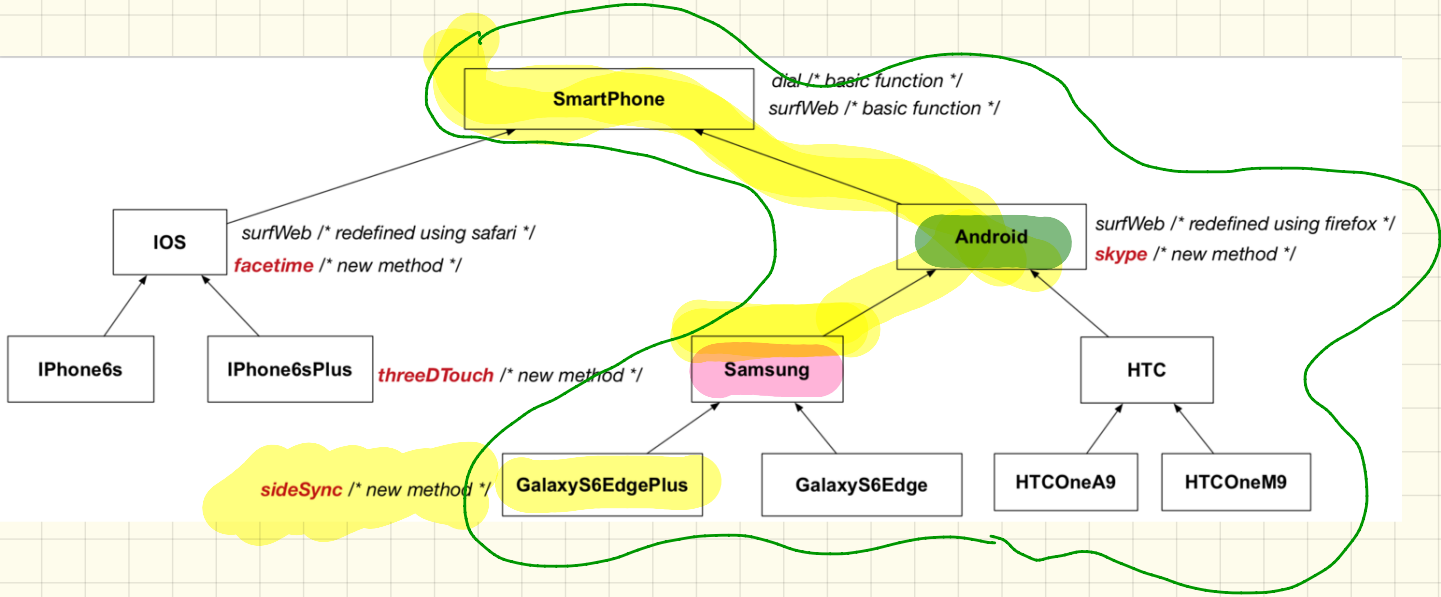


IP6s  $P = [ \quad \quad ]$

IP6sPlus  $P2 = (IP6sPlus) (P) \times$

ST  
IP6s

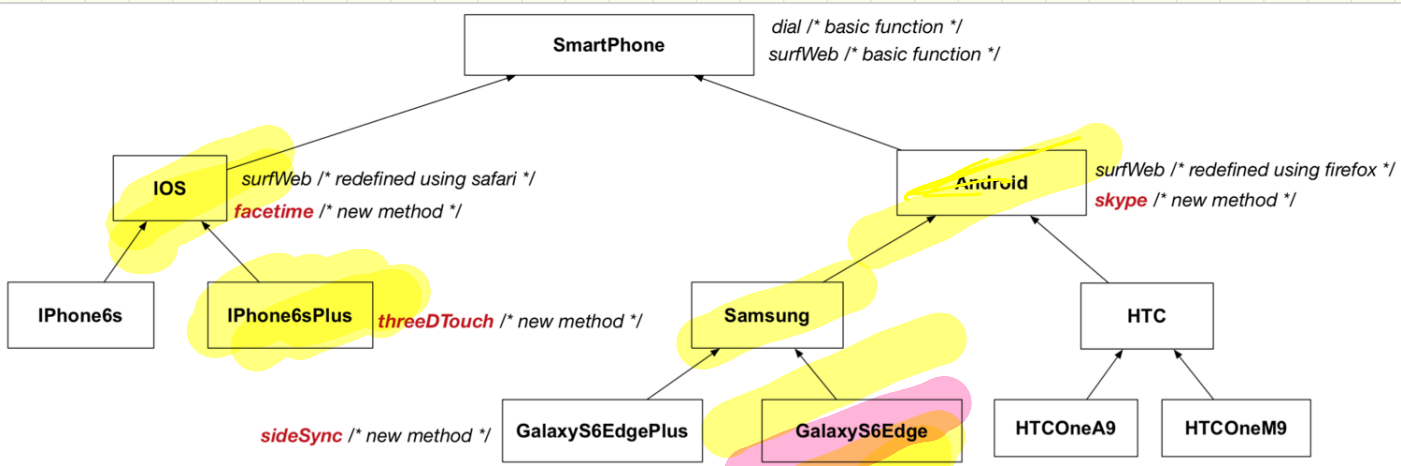


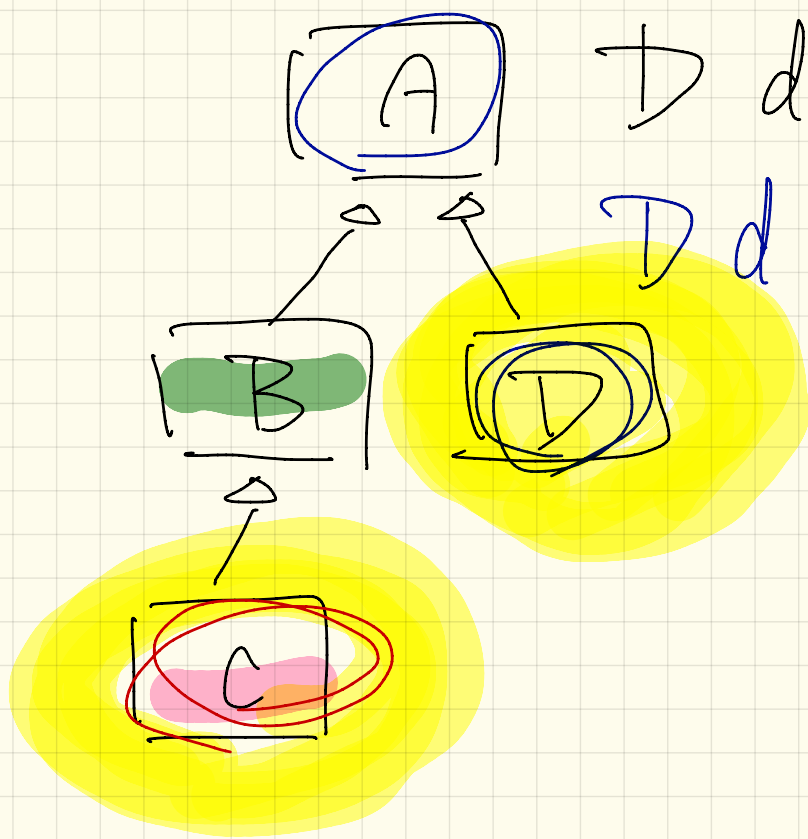


Android p = new Samsung();

① (Samsung) p

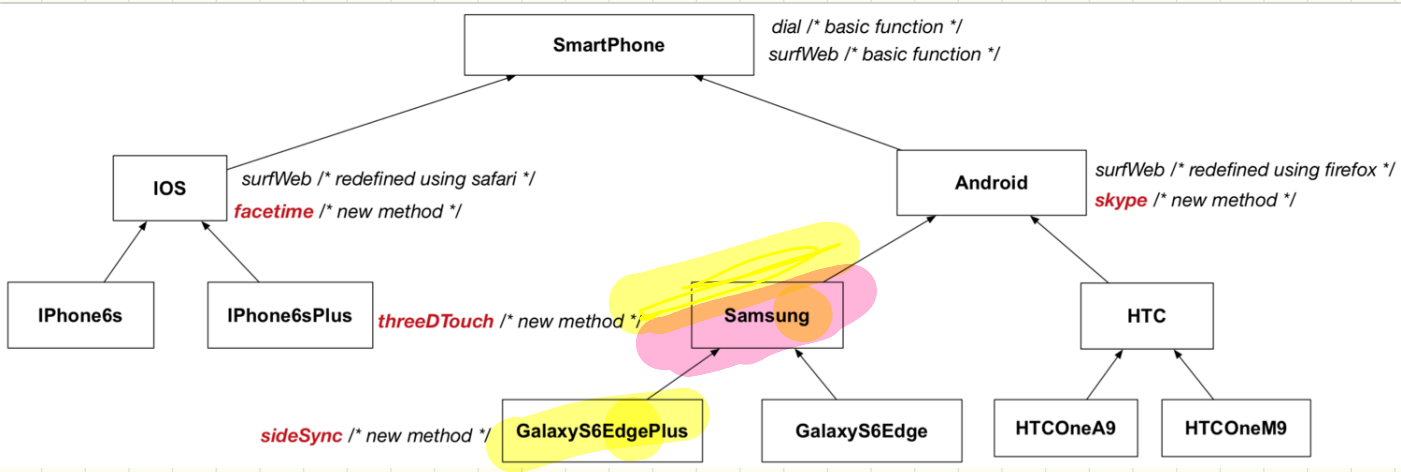
② (GalaxyS6EP) p





$$\vdash d = (D) b;$$

$$\vdash d = (D) (\underline{(A)} \underline{b})$$



declaration



Student [ ]

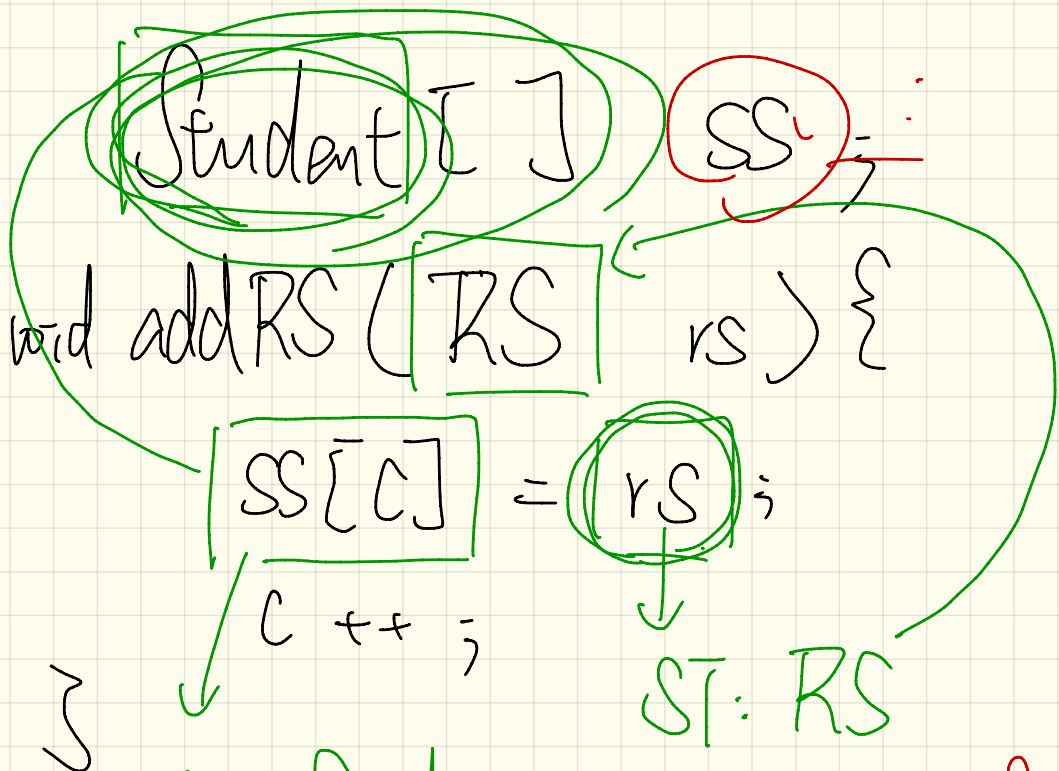
Student

S;

SS ;

① Static type of each item in  
SS is Student.

② dynamic type of each item in  
SS is Student.



```
SMS ( ) {  
    SS = new Student [10];  
}
```

```

class SMS {
    Student[] ss;
    void addStudent (Student s) {
        ss[0] = s;
    }
}

```

parameter

①  $rs = s$

②  $s = rs$

$rs$  →

$s$  →

rs

```

SMS sms = new SMS();
RS rs = new RS ("Rachael");
sms.addStudent (rs);

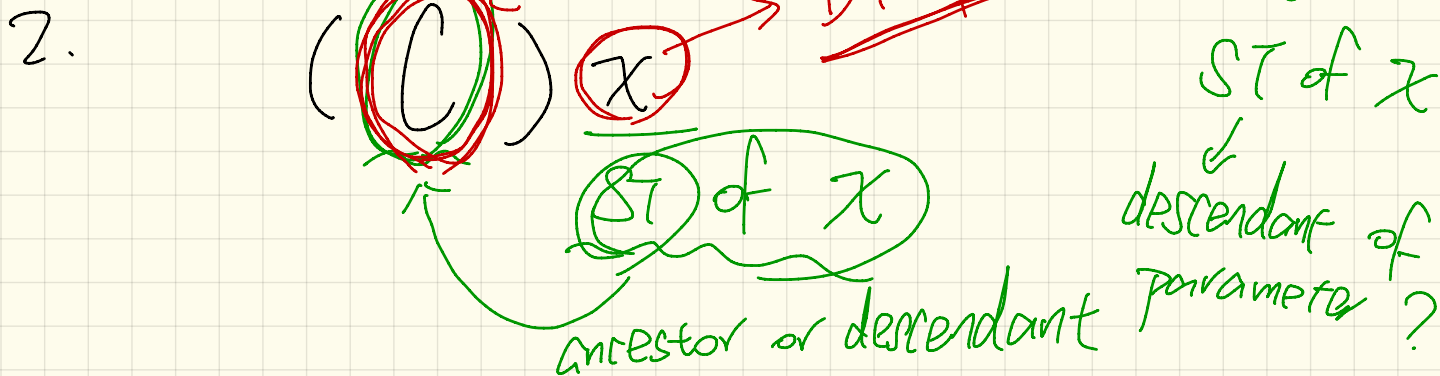
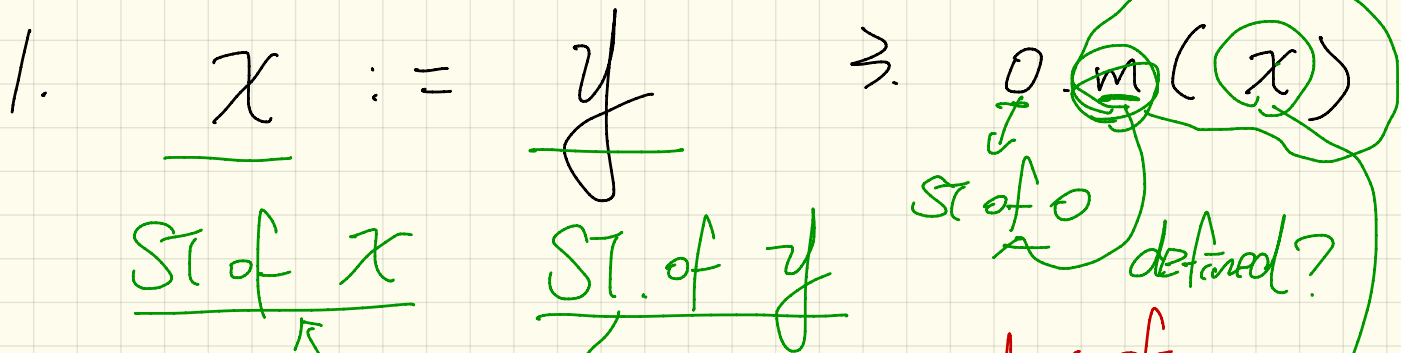
```

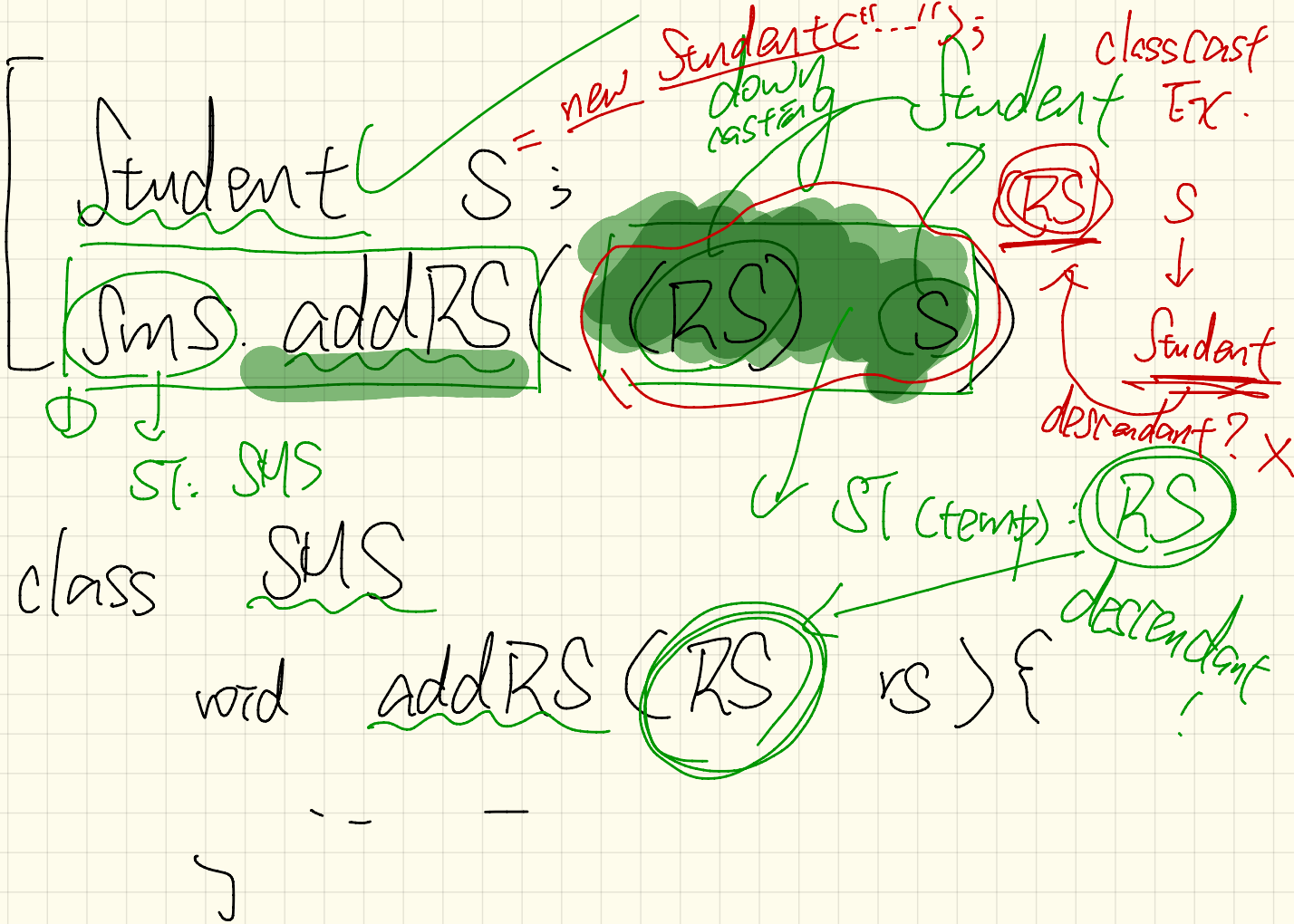
argument

Lecture 23

Tuesday Nov. 28







Student S = new Student();

ImS.addRS ((RS) S);

cast S to RS,

so that we can apply

expectation of (RS)

on S.

S.pr

pr

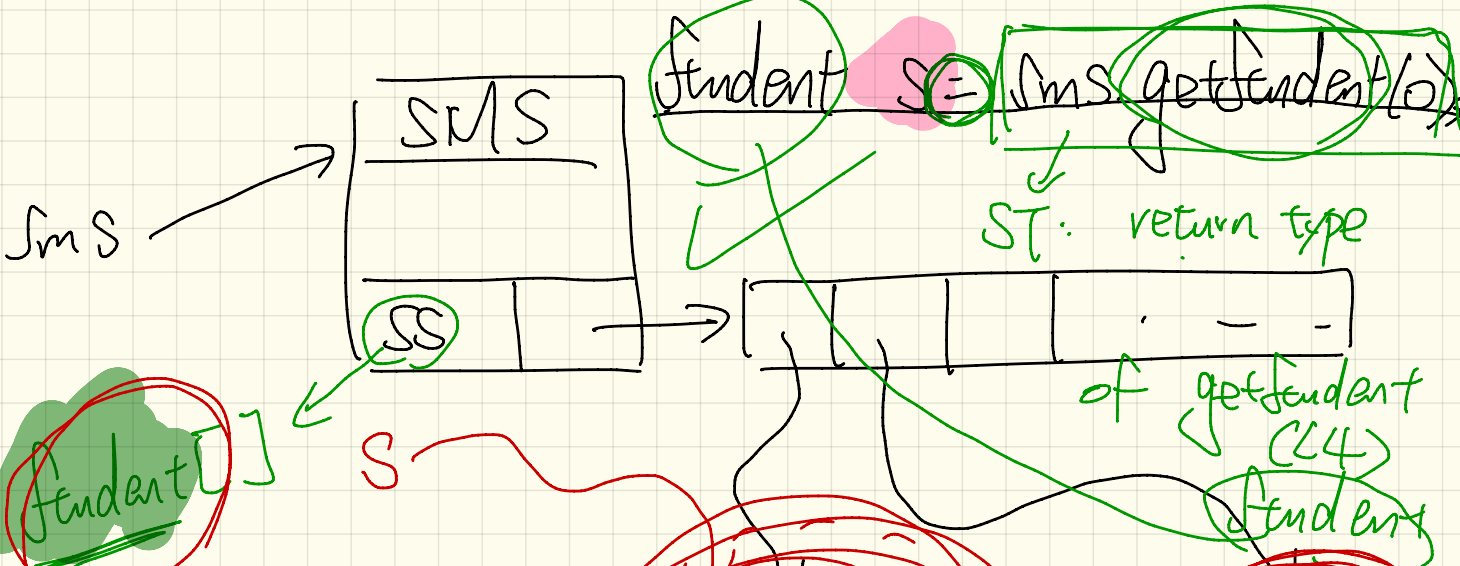
cannot meet exp.

↓

Class

cast

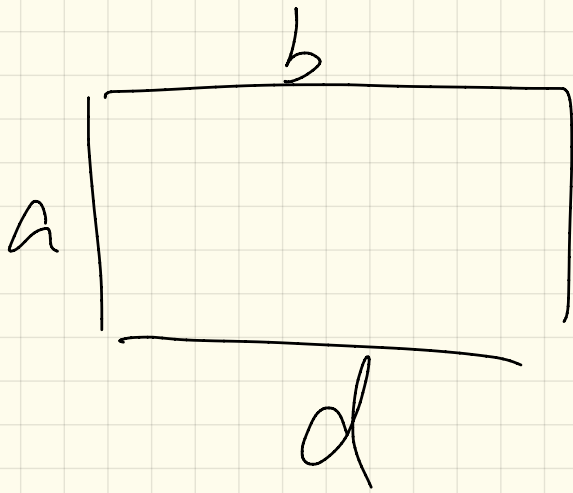
Exp.



`sms.addStudent(rs)`

```

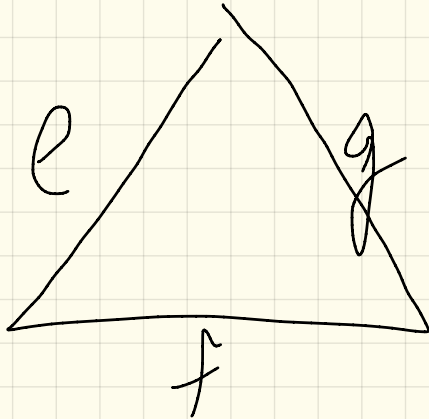
    ↳ sms.ss[0] = rs;
    sms.addStudent(nrs) → sms.ss[1] = nrs;
  
```



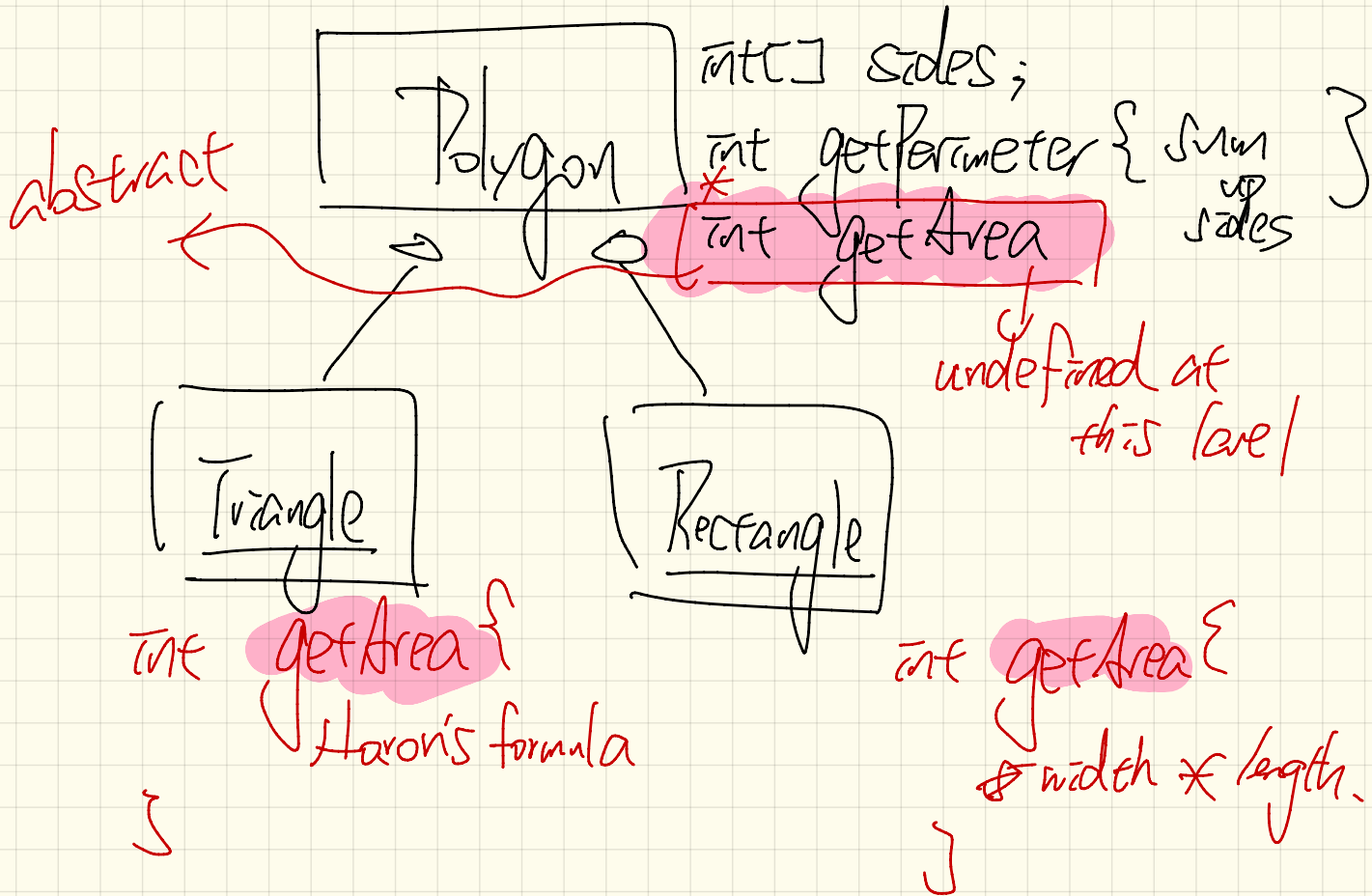
$$a + b + c + d$$

c  
int[] sides;

~~Perimeter~~  
Perimeter



$$e + g + f$$



~~Polygon~~

P = new

~~Polygon();~~

abstract class Polygon {

↓

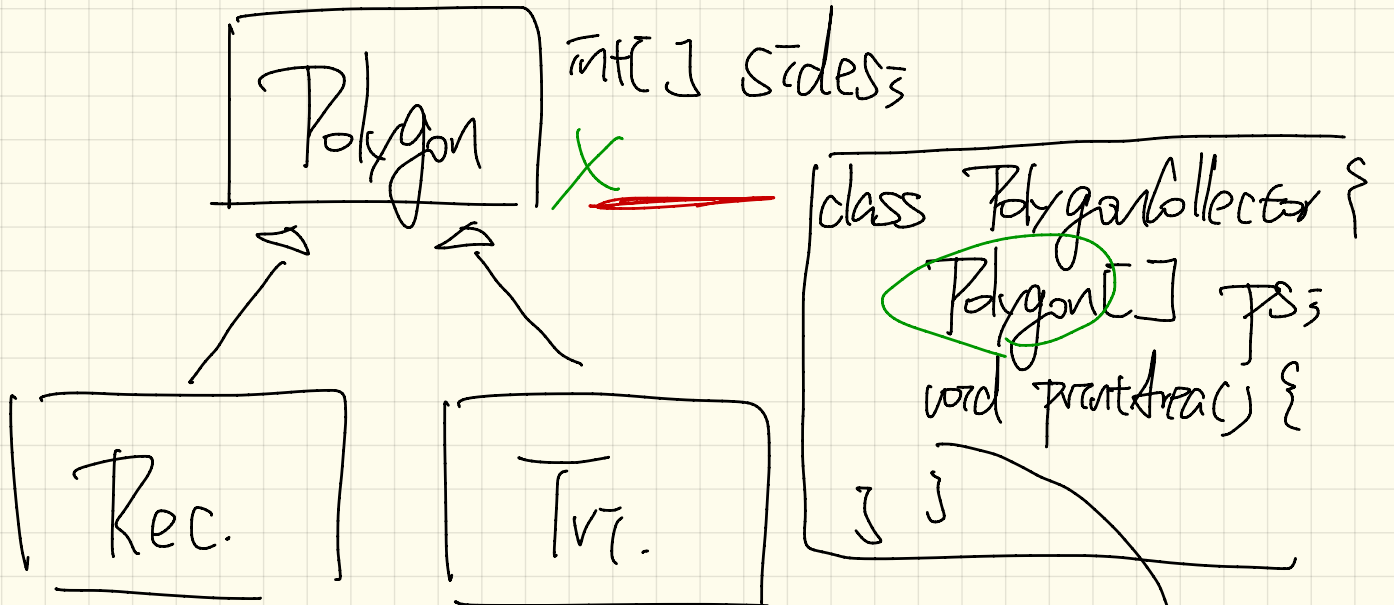
can't use

abstract class as DT.

↓

P. ~~getArea()~~ ?

crash :( undefined!

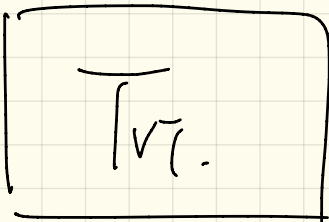


int sides

X

```

class PolygonCollector {
    Polygon[] ps;
    void printArea() {
    }
}
  
```



```

getArea() { -- }
  
```

```

getArea { -- }
  
```

getArea() not defined in

⇒ declare getArea() in Polygon (abstract)

```

for (Polygon p : ps) {
    p.getArea();
}
  
```



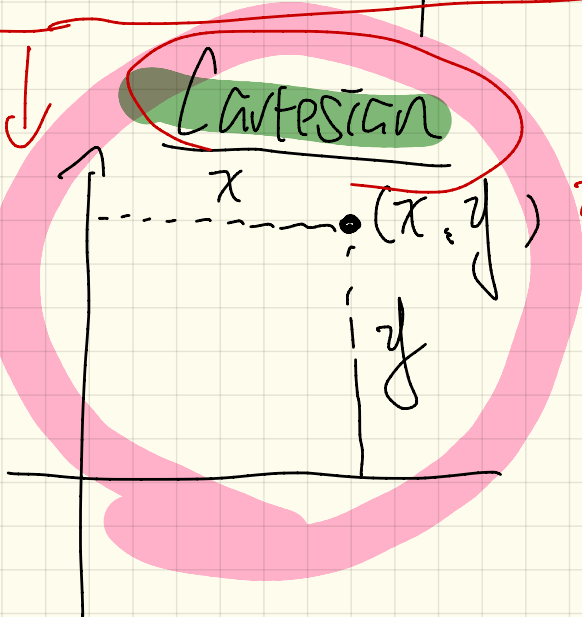
Lecture 24

Thursday Nov. 30

2-D points

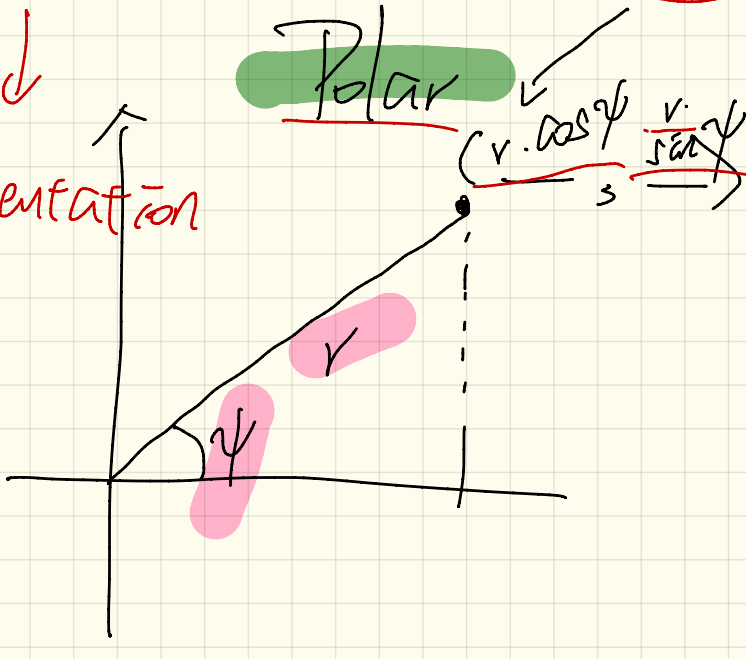
getX()  
getY()

CARTESIAN



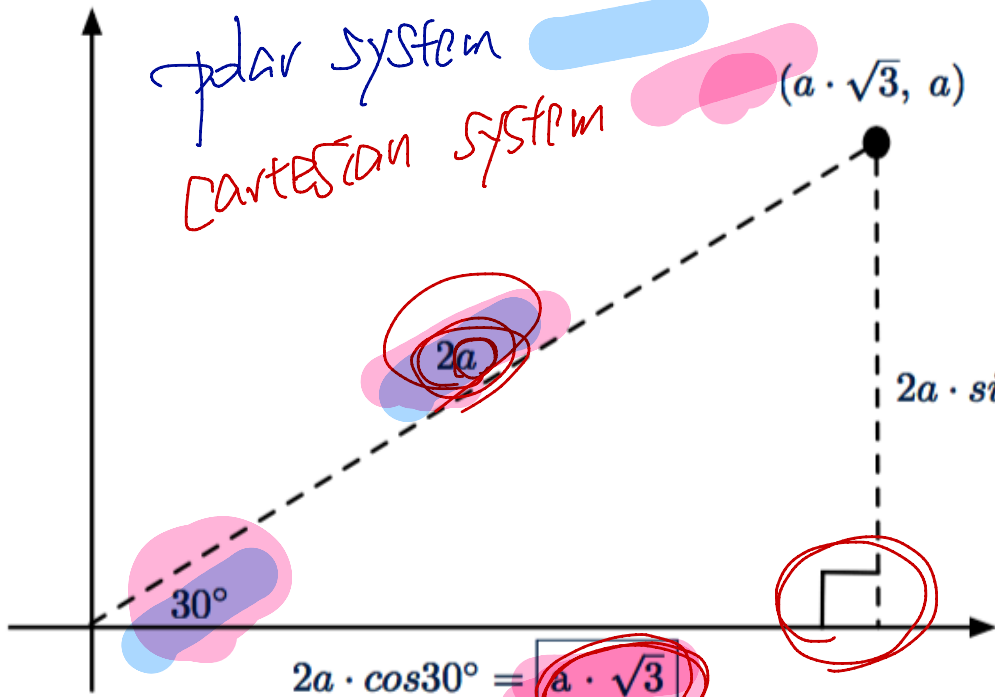
implementation

Polar



polar system

CARTESIAN SYSTEM



```
1 class Book {
2   String[] names;
3   Object[] records;
4   /* add a name-record pair to the book */
5   void add (String name, Object record) { ... }
6   /* return the record associated with a given name */
7   Object get (String name) { ... } }
```

```
1 Date birthday; String phoneNumber;
2 Book b; boolean isWednesday;
3 b = new Book();
4 phoneNumber = "416-67-1010";
5 b.add ("Suyeon", phoneNumber);
6 birthday = new Date(1975, 4, 10);
7 b.add ("Yuna", birthday);
8 isWednesday = b.get("Yuna").getDay() == 4;
```

ST: object

```

1 class Book {
2   String[] names;
3   Object[] records;
4   /* add a name-record pair to the book */
5   void add (String name, Object record) { ... }
6   /* return the record associated with a given name */
7   Object get (String name) { ... } }

```

Book (E)

unknown

Supplier

Book (Date)

converting E to led

User/Client

```

1 Date birthday; String phoneNumber;
2 Book b; boolean isWednesday;
3 b = new Book();
4 phoneNumber = "416-67-1010";
5 b.add ("Suyeon", phoneNumber);
6 birthday = new Date(1975, 4, 10);
7 b.add ("Yuna", birthday);
8 isWednesday = b.get("Yuna").getDay() == 4;

```

Date

descendant? X

ST: String

ST: Date

∴ only dates can be stored  
 ⇒ only dates are retrieved.

```

1  class Book {
2      String[] names;
3      Object [] records;
4      /* add a name-record pair to the book */
5      void add (String name, Object record) { ... }
6      /* return the record associated with a given name */
7      Object get (String name) { ... } }

```

Date  
~~Object~~ String

~~Object~~ Date

Date  
String

~~Object~~ String

Date Book <String> tb;

Book <Date> bd;

bd.get(...)  
tb.get(...)

ST? Date  
ST? String

① Node < String > n =

new Node < String > ("Tom", null);

② Node < String > n =

new Node < > (---);

END

ALL THE BEST !